Inelastic magnetic electron scattering M1 form factors in Ca-48 (M3Y fitting parameters consideration)

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Abstract

Inelastic magnetic electron scattering M1 at Ex =10.23 MeV form factors in Ca-48 have been investigated. The fp shell model space with four orbits and eight neutrons have been considered and FPD6 has been selected between 32 model space effective interactions to generates the model space vectors for the M1 transition with excitation energy Ex =10.23 MeV and for constructing OBDM. Discarded space (core and higher configuration orbits) has been included through the first order perturbation theory to couple the partice-hole pair of excitation in the calculation of the total M1 form factor and regarding the realistic interaction M3Y as a core polarization interaction with six sets of fitting parameters. Finally the theoretical calculations have been consisted with the experimental data for such transition form factor.

Keywords
M3Y, M1form factor electron scattering.

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دراسه عوامل التشكل الاستطاره الالكترونيه الغير المرنه المغناطيسيه في نواة Ca^{48} (أعتبارات معاملات الضبط لتفاعلات البقيه من نوع M3Y)

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الخلاصه:

تمت عمليه تحري عوامل التشكل للاستطاره الالكترونيه للانتقالات المغناطيسيه الغير المرنه M1 ذات طاقه التهيج E_x=10.23 MeV لنواة Ca^{48} أخذين بنظر الاعتبار القشره fp كأنموذج فضاء يحتوي ثمانيه نيوترونات و أربعه أغلفه ثانويه و أعتبار التفاعل المؤثر من نوع FPD6 كتفاعل فعال لتوليد دوال الموجه لأنموذج الفضاء سالف الذكر لدراسه عوامل التشكل للانتقالات M1 و حساب عنصر المصفوفه لكثافه أحتماليه أنتقال الجسيمه الواحده والمعرفه أختصارا OBDM . من خلال عمليه أستقطاب القلب (CORE POLARISATION) تم ضم الفضاء المستثنى (Discarded space) والذي يضم القلب الخامل و التشكلات العليا الى الحسابات بأستخدام نظريه الاضطراب من المرتبه الاولى والذي يتضمن ربط تفاعل (جسيم – فجوه) و بطاقات أستثاره مقدارها $2\hbar\omega$ و أعتماد تفاعلات البقيه من نوع M3Y-DD بواقع خمسة مجاميع من معاملات الضبط المستخدمه حديثا على النطاق الدولي و أخيرا تمت مقارنه الحسابات النظريه مع بعض القيم التجريبيه المتوفره .

Introduction

Inelastic electron scattering has regarded to be an excellent method for probing and measuring properties of the nuclei at their excited states, in particular their spins, parities, and the strength and structure of the transition operators connecting the ground and the excited states. The electron scattering method has certain unique advantage over other methods that indicate its continued use. The information available from experiments is certain to increase in quantity and quality[1].

By introducing a density-dependent contact term, M3Y-type interactions applicable to the core polarization calculations are used. In

order to view basic characters of the interactions with different types of fitting parameters, comparison has been carried out for the same transition multipolarity. This type of transitions had exhaust avast effort from the scientist where they were call it the mystre case

For Nuclei of A>40, the fp shell model space is the appropriate space, where a core of ^{40}Ca is assumed.

The theoretical study of such transition has also exhaust a huge efforts from the scientist in order to give the mystery case a true physical interpretations, so the study of such case shall vary from one to other depending on the theoretical framework that he depends on. Random Phase Approximation (RPA) technique in the 0ħω configuration space had been used for the studying of magnetic dipole transition in (^{48}Ca) [2, 3] and it was dominated by the neutron $1f^{-1}_{7/2}1f_{5/2}$ particle-hole configuration, Pure 1p-1h RPA excitations, meson exchange currents[4], the Quenching of the (e,e') form factor of the M1 transition to the 10.23 MeV state in ^{48}Ca [5], and electroexcitation of magnetic states in ^{48}Ca by the use of electron scattering and RPA [6] but poor agreement had reflected, so the use of (SRPA + ρ-meson exchange) [7] to study the B(M1) transition by the use of the technique between the two particle two hole states with (4ħω) excitation, these results are in a good agreement with the experimental data. Calculation in the complete $1f_{7/2}1f_{5/2}2p_{3/2}2p_{1/2}$ model space were presented for the M1 excitation of the ground states in ^{48}Ca have been carried out by the use of nuclear shell model wave function with energy function [8] Shell-model plus Hartree - Fock calculations for the neutron-rich Ca isotopes have been studied [9] including comparative study with the result of experimental inelastic electron scattering form factor. Inelastic electron scattering form factor have been investigated for ^{48}Ca. The investigations have been performed in terms of the configuration mixing shell model with limiting number of orbital in the model space, outside the inert core. The discarded space has been included, which is called core-polarization effects, through a microscopic theory which considers a particle-hole excitations from the core orbits and from the model space orbits into the higher orbits with 2ħω excitations, the two body Michigan sum of three range Yukawa potential (M3Y-Elliote fitting) interaction is used for the core polarization matrix elements. The simple harmonic oscillator potential is used to generate the single particle wave functions, where an analytical solution is possible the results are in a good agreement with the experimental data[10].

The aim of this paper is to use a realistic effective nucleon-nucleon (NN) M3Y interaction with six sets of fitting parameters as a residual interaction to calculate the core polarization (CP) effects through a microscopic theory, with a selection of model space effective interaction which generates the model space wave functions(shell model wave functions) and highly excited states. Harmonic oscillator wave functions to be adopted as a single particle wave function. Michigan sum of three-range Yukawa potential (M3Y) interaction of Berstch [11] is adopted as a residual interaction for the core polarization matrix elements. A computer program in FORTRAN 90 language is developed to include fp-shell in the original code [12],
which calculates the model space form factors (zeroth-order) and the first-order cp effects.

Theory

The reduced matrix element of the electron scattering operator $T\Lambda$ is expressed as the sum of the product of the matrix elements of the one-body density matrix (OBDM) $X^{\Lambda}_{\Gamma_f\Gamma_i}(\alpha,\beta)$ times the single-particle matrix elements, and is given by [13]:

$$\langle \Gamma_f \| \hat{T}_\Lambda \| \Gamma_i \rangle = \sum_{\alpha\beta} X^{\Lambda}_{\Gamma_f \Gamma_i}(\alpha,\beta) \langle \alpha \| \hat{T}_\Lambda \| \beta \rangle$$

……. (1)

where α and β label single-particle states (isospin is included) for the shell model space. The states $|\Gamma_i|)$ and $|\Gamma_f|)$ are described by the model space wave functions. Greek symbols are used to denote quantum numbers in coordinate space and isospace, i.e., $\Gamma i \equiv JiTi$, $\Gamma f \equiv Jf\ Tf$ and $\Lambda \equiv JT$. According to the first-order perturbation theory, the single-particle matrix element of the one-body operator is given by [13].

$$\langle \alpha \| \hat{T}_\Lambda \| \beta \rangle = \langle \alpha \| \hat{T}_\Lambda \| \beta \rangle + \left\langle \alpha \left| \left\| \hat{T}_\Lambda \frac{Q}{E_i - H_Q} V_{res} \right\| \right| \beta \right\rangle + \left\langle \alpha \left| \left\| V_{res} \frac{Q}{E_f - H_Q} \hat{T}_\Lambda \right\| \right| \beta \right\rangle \ldots\ldots\ldots\ldots\ldots (2)$$

The first term is the zero-order contribution. The second and third terms are the first-order contributions which give the higher-energy configurations (hec). The operator Q is the projection operator onto the space outside the model space. The hec terms given in Eq. (2) are written as [13],

$$\sum_{\alpha_1\alpha_2\Gamma} \frac{(-1)^{\beta+\alpha_2+\Gamma}}{e_\beta - e_\alpha - e_{\alpha_1} + e_{\alpha_2}} (2\Gamma+1) \begin{Bmatrix} \alpha & \beta & \Lambda \\ \alpha_2 & \alpha_1 & \Gamma \end{Bmatrix} \sqrt{(1+\delta_{\alpha_1\alpha_2})(1+\delta_{\alpha_2\beta})} * \langle \alpha\alpha_1 | V_{res} | \beta\alpha_2 \rangle \langle \alpha_2 \| \hat{T}_\Lambda \| \alpha_1 \rangle + \text{term with } \alpha_1 \text{ and } \alpha_2 \text{ exchanged with an over all minus sign}$$

……. (3)

where the index α_1 runs over particle states and α_2 over hole states and e_i is the single-particle energy, and is calculated according to [13] as,

$$e_{nlj} = (2n+l-\frac{1}{2})\hbar\omega + \begin{cases} -\frac{1}{2}(l+1)\langle f(r)\rangle_{nl} & for\ j = l-\frac{1}{2} \\ \frac{1}{2} l \langle f(r)\rangle_{nl} & for\ j = l+\frac{1}{2} \end{cases} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

With

$$\langle f(r)\rangle_{nl} \approx -20A^{-2/3} MeV$$

$$\hbar\omega = 45A^{-1/3} - 25A^{-2/3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The single particle matrix elements reduced in both spin and isospin, are written in terms of the single-particle matrix elements reduced in spin only [13].

$$\langle A \| \hat{T}_\Lambda \| \beta \rangle = \sqrt{\frac{2T+1}{2}} \sum_{t_z} I_T(t_z) \langle j_2 \| \hat{T}_\Lambda \| j_1 \rangle \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$I_T(t_z) = \begin{cases} 1 & for\ T = 0 \\ (-1)^{\frac{1}{2}-t_z} & for\ T = 1 \end{cases} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Where $t_z = \frac{1}{2}$ for proton and -1/2 for neutron. Higher energy configurations are taken into consideration through 1p–1h excitations from the model space orbits into higher orbits. All excitations are considered with $2\hbar\omega$ excitations.

For the residual two-body interaction *V*res, the M3Y interaction of Nakada. [15] is adopted. The form of the potential is defined in Eq. (2) in Ref. [15]. The parameters of 'Paris'(P0) and Rieds five sets (P1, P2, P3, P4, P5) are used which are given in used to get the relation between the two-body shell model matrix elements and the relative and the center of mass coordinates, using the harmonic oscillator radial wave functions with Talmi–Moshinsky transformation.

Electron scattering form factor involving angular momentum *J* and momentum transfer *q*, between initial and final nuclear

shell model states of spin *Ji,f* and isospin *Ti,f* are [13]

$$\left|F_J^{\eta}(q)\right|^2 = \frac{4\pi}{Z^2(2J_i+1)}$$

$$\left| \sum_{T=0,1} (-1)^{T_f - T_{Zf}} \begin{pmatrix} T_f & T & T_i \\ -T_{Zf} & M_T & T_{Zi} \end{pmatrix} \left\langle \Gamma_f \left\| T^{\eta}_{J,T}(q) \right\| \Gamma_i \right\rangle \right|^2$$

$$\times \left|F_{c.m}(q)\right|^2 \times \left|F_{f.s}(q)\right|^2 \quad \text{.....(8)}$$

The parameters with the superscript and subscritions (the tees) are the fitting parameters extracted from real nucleon-nucleon interactions and table (2) will shows these parameters for each version of M3Y interaction.

$$v_{12} = v_{12}^{(c)} + v_{12}^{(LS)} + v_{12}^{(TN)} + v_{12}^{(DD)}$$

.....(9)

where

$$v_{12}^{(c)} = \sum_n (t_n^{(SE)} P_{SE} + t_n^{TE} P_{TE} + t_n^{(SO)} P_{SO} + t_n^{(TO)} P_{TO}) f_n^{(C)}(r_{12})$$

$$v_{12}^{(LS)} = \sum_n (t_n^{(LSE)} P_{TE} + t_n^{(LSO)} P_{TO}) f_n^{(LS)}(r_{12}) L_{12}.(\vec{s}_1 + \vec{s}_2)$$

$$v_{12}^{(TN)} = \sum_n (t_n^{(TNE)} P_{TE} + t_n^{(TNO)} P_{TO}) f_n^{(TN)}(r_{12}) r_{12}^2 S_{12}$$

$$v_{12}^{(DD)} = t^{(DD)} \{(1 - x^{(DD)}) P_{SE} + t_n^{(DD)} (1 + x^{(DD)}) P_{TE})\} \delta^{(TN)}(r_{12}).$$

.....(10)

Results discussion and conclusions.

Semirealistic nucleon-nucleon interactions applicable to the self-consistent mean-field (both Hartree-Fock and Hartree-Fock-Bogolyubov) calculations were developed by modifying the M3Y interaction. The modification is made to reproduce binding energies and rms matter radii of doubly magic nuclei, single-particle levels in 208Pb and even-odd mass differences of the Sn isotopes. They found parameter sets with and without the tensor force. The new interactions were further checked by the saturation properties of the uniform nuclear matter, including the Landau-Migdal parameters [15].these sets of new versions of M3Y the realistic are used to be as a residual interaction for the core polarization in order to be the remaining space which is suppose to enhance the total results for the form factors in addition to the model space. Table(1) shows the values of OBDM for the third M1 excited state.

Table(1), values of OBDM for the third M1 excited state.

J_i	J_f	OBDM (ΔT=0)	OBDM(ΔT=1)
7/2	7/2	-٠,٠٠٣٣٢	-٠,٠٠٢١٤
7/2	5/2	٠,٠٢٦٠٣	٠,٠١٦٨٠
3/2	3/2	٠,٠٠٦٤٩	٠,٠٠٤١٩
3/2	5/2	-٠,١٥٤١٧	-٠,٠٩٩٥١
3/2	1/2	-٠,٠٦٤٣٢	-٠,٠٤١٥٢
5/2	7/2	٠,٢٣٠٧٠	٠,١٤٨٩١
5/2	3/2	-٠,١٧٧١٤	-٠,١١٤٣٤
5/2	5/2	٠,٠٠٢٩٩	٠,٠٠١٩٣
1/2	3/2	-٠,١٣٥٦٧	-٠,٠٨٧٥٧
1/2	1/2	-٠,٠٠٧٨١	-٠,٠٠٥٠٤

Table (2) shows the values of the fitting parameters for five versions of density dependent M3Y which are (M3Y-P1, P2, P3, P4, P5) beside the earlier version of density independent M3Y with Paris (M3Y-P0) [16] and (M3Y-E) Elliott fitting [17]. Density dependent M3Y which is (M3Y-P1) had been proven to be the most suitable realistic interaction to study C6 form factors in Ti-50 [18], in comparison with density independent M3Y with Paris (M3Y-P0) [16] and (M3Y-E) Elliotte fitting [17] where the core polarization part were in negative contribution with the model space one for the same case (C6 form factors in Ti-50) [10], but in general the residual interaction density independent M3Y

had the same behavior for another cases resulted in ref. [10] concluded when we had used MSDI interaction for the same purpose in the same, from fig.(1), fig.(2), fig.(3), fig.(4), fig.(5), fig.(6)
the model space contribution coincides with the experimental data very exact but the core polarization one does not, both in form factor and momentum values so the total form factor for such transition will be expected to be not coincides for every types of M3Y and this results is as same as that resulted in ref. [10], and the residual interaction does not modifies the core part in contra behavior if we use it to calculate the core part in longitudinal form factors [10, 18]. In this case we conclude that the core ^{40}Ca is inert and there are no excited particles out of the model space which has eight neutrons and four active orbits enough to be predominant to construct the general behavior for M1 form factors at excitation energy (E_x=10.23 MeV) which is the mystery case, in another hand this case might probably used to distinguish between the six versions as a function of momentum transfer, so figs. (7), (8) show the comparison for the core parts and the total form factor respectively. And it is clear that they are different between each other in cases of diffraction pattern, diffraction minima, shifting along the q-direction, and their maximum form factors values, so it is clear that the fitting parameters have a large effect on the total behavior on the calculation and results for inelastic transverse electron scattering M1 form factors and we would like to say that the for inelastic transverse electron scattering M1 form factors as a probe to trace the difference between these five types

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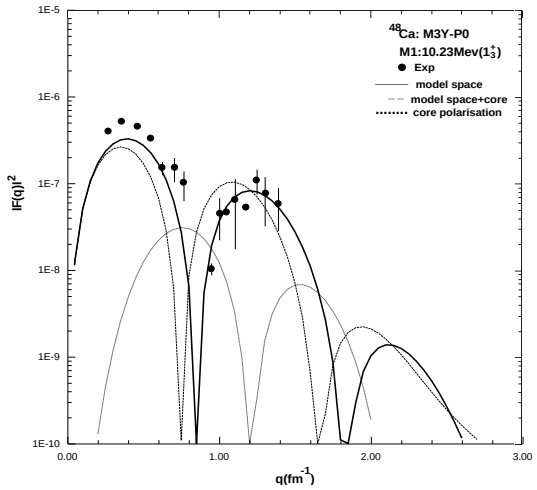

Fig. 1 Inelastic transverse electron scattering M1 form factor with M3Y-P0.

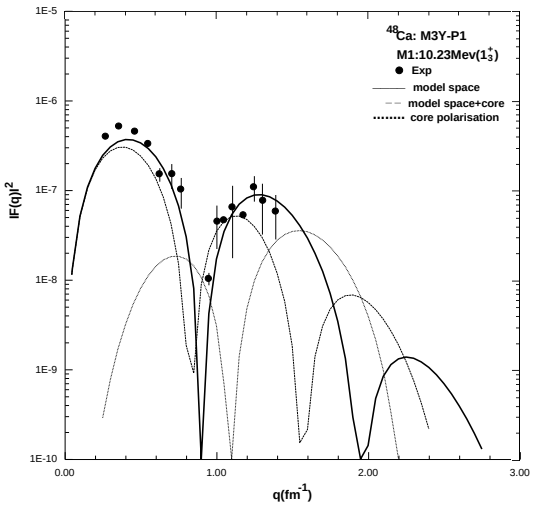

Fig. 2 Inelastic transverse electron scattering M1 form factors with M3Y-P1

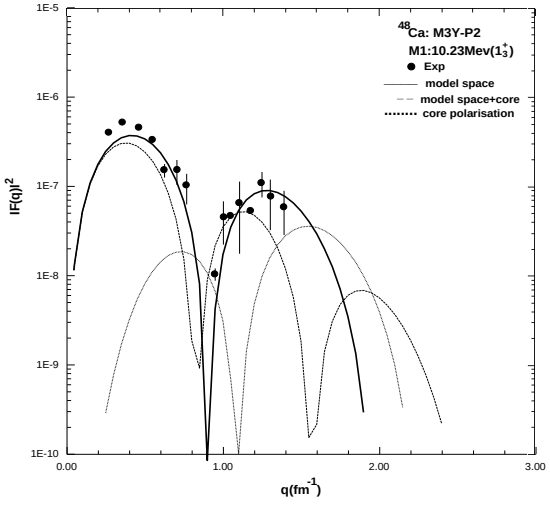

Fig. 3 Inelastic transverse electron scattering form factors with M3Y-P2.

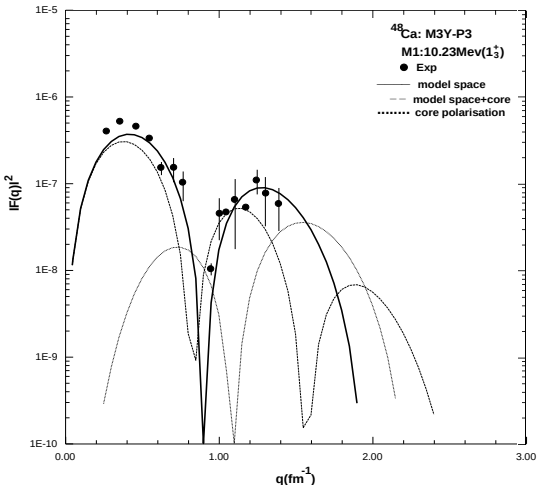

Fig. 4 Inelastic transverse electron scattering M1 form factors with M3Y-P3.

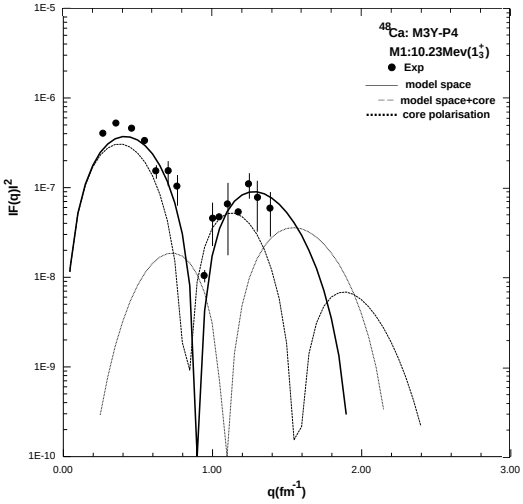

Fig. 5 Inelastic transverse electron scattering M1 form factors with M3Y-P4.

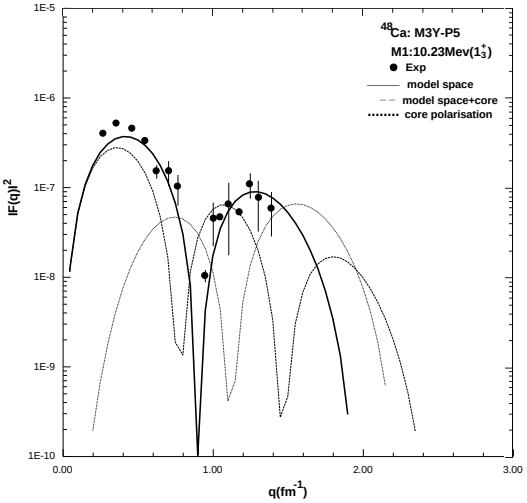

Fig.e 6 Inelastic transverse electron scattering M1 form factors with M3Y-P3

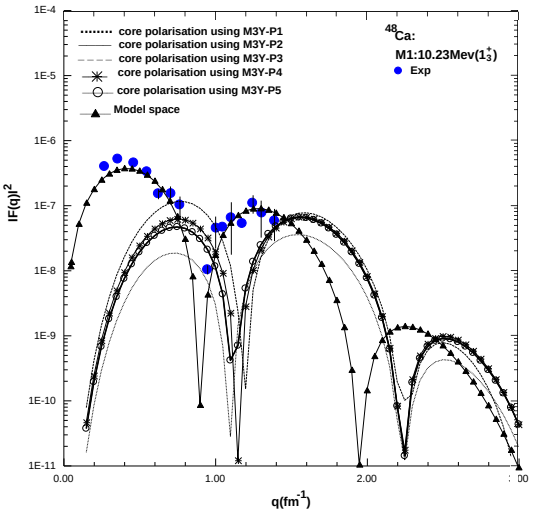

Fig. 7 Comparison between the five versions of density dependent M3Y, core parts.

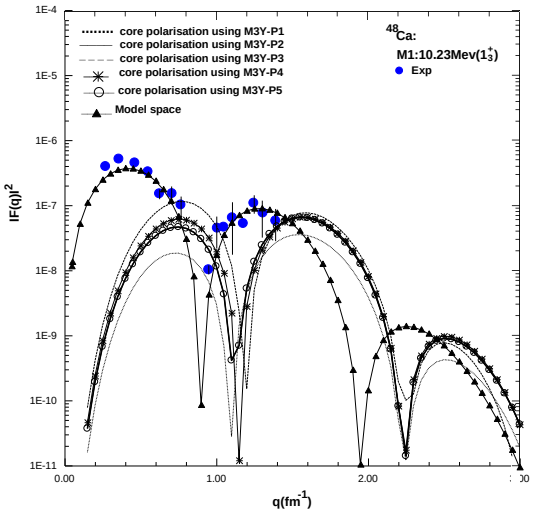

Figure 8 comparison between the five versions of density dependent M3Y, total M1 form factors

Table (1). Shows the values of the best fit to the potential parameters [15].

Parameters	unit	M3Y-E*	M3Y-P0	M3Y-P1†	M3Y-P2	M3Y-P3	M3Y-P4	M3Y-P5
$R_1^{(C)}$	(fm)	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$t_1^{(SE)}$	(MeV)	9958	11466.	8599.5	8027.	8027.	8027.	8027.
$t_1^{(TE)}$	(MeV)	11849	13967.	10475.25	6080.	7130.	5503.	5576.
$t_1^{(SO)}$	(MeV)	26941	−1418.	−1418.	−11900.	−1418.	−12000.	−1418.
$t_1^{(TO)}$	(MeV)	0.0	11345.	11345.	3800.	11345.	3700.	11345.
$R_2^{(C)}$	(fm)	0.40	0.40	0.40	0.40	0.40	0.40	0.40
$t_2^{(SE)}$	(MeV)	-3105	−3556.	−3556.	−2880.	−2637.	−2637.	−2650.
$t_2^{(TE)}$	(MeV)	-3761	−4594.	−4594.	−4266.	−4594.	−4183.	−4170.
$t_2^{(SO)}$	(MeV)	-2777	950.	950.	2730.	950.	4500.	2880.
$t_2^{(TO)}$	(MeV)	0.0	−1900.	−1900.	−780.	−1900.	−1000.	−1780.
$R_3^{(C)}$	(fm)	1.414	1.414	1.414	1.414	1.414	1.414	1.414
$t_3^{(SE)}$	(MeV)	-10.463	−10.463	−10.463	−10.463	−10.46	−10.46	−10.463

						3	3	
$t_3^{(TE)}$	(MeV)	-10.463	−10.463	−10.463	−10.463	−10.463	−10.463	−10.463
$t_3^{(SO)}$	(MeV)	31.389	31.389	31.389	31.389	31.389	31.389	31.389
$t_3^{(TO)}$	(MeV)	3.488	3.488	3.488	3.488	3.488	3.488	3.488
$R_1^{(LS)}$	(fm)	0.25	0.25	0.25	0.25	0.25	0.25	0.25
$t_1^{(LSE)}$	(MeV)	0.0	−5101.	−9181.8	−9181.8	−10712.1	−8671.7	−11222.2
$t_1^{(LSO)}$	(MeV)	-2672	−1897.	−3414.6	−3414.6	−3983.7	−3224.9	−4173.4

(Continue)

Table (2). Shows the values of the best fit to the potential parameters [15].

$R_2^{(LS)}$	$R_2^{(LS)}$(fm)	0.40	0.40	0.40	0.40	0.40	0.40	0.40
$t_2^{(LSE)}$	$t_2^{(LSE)}$(MeV)	-813.0	−337.	−606.6	−606.6	−707.7	−572.9	−741.4
$t_1^{(LSO)}$	(MeV)	-2672	−1897.	−3414.6	−3414.6	−3983.7	−3224.9	−4173.4
$R_2^{(LS)}$	(fm)	0.40	0.40	0.40	0.40	0.40	0.40	0.40
$t_2^{(LSE)}$	(MeV)	-813.0	−337.	−606.6	−606.6	−707.7	−572.9	−741.4
$t_2^{(LSO)}$	(MeV)	-620.0	−632.	−1137.6	−1137.6	−1327.2	−1074.4	−1390.4
$R_3^{(LS)}$	(fm)	1.414	1.414	1.414	1.414	1.414	1.414	1.414
$t_3^{(LSE)}$	(MeV)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$t_3^{(LSO)}$	(MeV)	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$R_1^{(TN)}$	(fm)	0.4	0.4	0.4	0.4	0.4	0.4	0.4
$t_1^{(TNE)}$	(MeVfm^{-2})	0.0	−1096.	−131.52	−131.52	−1096.	0.0	−1096.
$t_1^{(TNO)}$	(MeVfm^{-2})	0.0	244.	29.28	29.28	244	0.0	244.
$R_2^{(TN)}$	(fm)	0.70	0.70	0.70	0.70	0.70	0.70	0.70
$t_2^{(TNE)}$	(MeVfm^{-2})	-171.7	−30.9	−3.708	−3.708	−30.9	0.0	−30.9
$t_2^{(TNO)}$	(MeVfm^{-2})	283.0	15.6	1.872	1.872	15.6	0.0	15.6
$R_3^{(TN)}$	(fm)	1.414	1.414	1.414	1.414	1.414	1.414	1.414
$t_3^{(TNE)}$	(MeVfm^{-2})	-78.03	0.0	0.0	0.0	0.0	0.0	0.0
$t_3^{(TNO)}$	(MeVfm^{-2})	13.62	0.0	0.0	0.0	0.0	0.0	0.0
$t_{DD}^{(SE)}$	(MeV fm)	0.0	0.0	1092	181.	220.	248.	126.
$t_{DD}^{(TE)}$	(MeV fm)	0.0	0.0	1331	1139.	1198.	1142.	1147.