Iraqi Journal of Physics, 2010 **Vol. 8, No.13, PP. 45 -52**

Partial Level Densities for Neutron Induced Pre-equilibrium Nuclear Reactions

Mahdi Hadi Jasim, Shafik Shaker Shafik and Shaimaa Mahdi Qaduri

Department of Physics, College of Science, University of Baghdad, Jadiriya, Baghdad, Iraq

Abstract

The state and partial level densities were calculated using the corresponding formulas that are obtained in the frame work of the exciton model with equidistant spacing model (ESM) and non-ESM (NESM). Different corrections have been considered, which are obtained from other nuclear principles or models. These corrections are Pauli Exclusion Principle, surface effect, pairing effect, back shift due to shell effect and bound state effect . They are combined together in a composite formula with the intention to reach the final formula. One-component system at energies less than 100 MeV and mass number range (50-200) is assumed in the present work. It was found that Williams' plus spin formula is the most effective approach to the composite formula, and it is in good agreement with experimental results. All calculation has been made using programs with MATLAB language written for this purpose.

Keywords
partial level densities

Article info
Received: Mar. 2010
Accepted: Apr. 2010
Published: Dec. 2010

كثافات المستوي الجزئية للنيترون التي تحدث تفاعلات نووية شبه متوازنة

مهدي هادي جاسم, شفيق شاكر شفيق, شيماء مهدي قدوري

قسم الفيزياء, كلية العلوم, جامعة بغداد/ بغداد-العراق

الخلاصة

تم حساب كثافات المستوي الجزئية باستخدام الصيغ المعتمدة ضمن إطار أنموذج الجسيمة المتهيجة مع الأخذ بنظر الاعتبار تساوي المسافات بين المستويات وعدم تساويها. أخذت بنظر الاعتبار تصحيحات مختلفة اعتمادا على المبادئ والمفاهيم النووية النظرية التي تعتمد في النماذج الأخرى. من بين هذه التصحيحات: مبدأ الاستبعاد لباولي, تأثير السطح, الازدواج بالإضافة إلى تأثير أنموذج الاغلفة و الحالة المقيدة للطاقة العظمى للجسيمة. ركبت هذه التصحيحات معا في صيغة واحدة بهدف الوصول إلى صيغة تتضمن أهم التصحيحات. استخدمت المعالجة الحالية نظام المركبة الواحدة بطاقات اقل من 100 MeV ومدى عدد كتلي 200-50 . عند المقارنة بين نتائج كثافات المستوي الجزئية المحسوبة من جميع الصيغ السابقة وجد إن الصيغة التي تتضمن حد الاستبعاد لباولي وتوزيع البرم هي الأقرب إلى الصيغة المركبة لذلك تم استخدامها في حساب الكثافة الكلية للمستوي ثم مقارنتها مع النتائج العملية, حيث كانت تتفق معها. حسبت كثافة المستوي الجزئية بواسطة البرامج المكتوبة بلغة MATLAB في الدراسة الحلية.

Introduction

The properties of nuclei at high excitation energies are important for many nuclear reactions, particularly those that pass through a highly excited compound nucleus. It is sufficient for this purpose to know the over-all statistical properties of the nuclear levels, i.e. the probability distribution functions of the parameters of the nuclear levels as a function of excitation energy. The nuclear level density (PLD) represents the most important property than others, especially in the cross section calculation of compound nucleus and pre-equilibrium model reactions. The level density can be divided according to the excitation energy into two regions, namely ; the low and high energy excitations. The low-lying nuclear excited levels are small in number, well separated, and rather than simple in structure [1]. With increasing excitation energy (E), beyond a few MeV, the nature of the excitation levels becomes very complicated where the spacing between the levels is progressively reduced [1]. However, an individual description of the discrete levels becomes impossible. Indeed, not only the levels are more and more close to each other, but acquire larger widths. Therefore, the only possibility to describe them is the statistical frame work [2].

II. State Density with ESM Formulas.

The first attempts to study level densities have been achieved at the end of the thirties, where the ESM was suggested by Bethe, (1936) [3]. In this model, the nucleus is represented as a system of fermions susceptible to occupy the levels with density sometimes described as the Fermi gas level density expression, and this is incorrect. In a Fermi gas, the single particle level density increases approximately as the square root of the particle kinetic energy, while in the model of the present study it is a constant. This expression stands to the zeroth order approximation of the level density. Therefore, the single particle levels (g) are equidistant and no degenerate with a constant single particle level spacing of (D=1/g). An analytical formula of cumulated densities based on saddle point approximation was obtained by Bethe in 1937 [2]. The simplest formula to calculate the PLD is obtained by Griffin [4].
Griffin [4].

$$W(n,E)=\frac{g^{n}E^{n-1}}{n!(n-1)!} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where n is exciton number. Eq.(1) can be reformulated [5] as:

$$W(n,E)=W(p,h,E)=\frac{g^{n}E^{n-1}}{p!h!(n-1)!} \quad ..(2)$$

which is called Ericson formula, where p is number of particles and h is the number of holes. However, to make PLD (eq. (2)) more realistic, one must add some corrections which can be listed as follows:

a- PLD with Pauli Effect Formula:

In the language of pre equilibrium models, the Pauli principle requires that no two excitons of the same type are allowed to be in the same state, which implies that they cannot have the same energy. Then, the PLD takes the form [6]:

$$\alpha_{Ph}=\frac{1}{2g}[(p^{2}+p)+(h^{2}-h)] \quad \ldots\ldots\ldots.(3)$$

where Θ is the Heaviside step function which is unity for positive values of the argument and zero otherwise, and
:

$$W(p,h,E)=\frac{g^{p+h}(E-A_{Ph})^{p+h-1}}{p!h!(p+h-1)!}\Theta(E-\alpha_{Ph})..(4)$$

which is the minimal energy needed to put (p) particles and (h) holes in the levels taking into account the Pauli effect. The Pauli correction term, which lowers the energy in eq. (3), is:

$$A_{Ph}=\frac{1}{4g}[(p^{2}+p)+(h^{2}-3h)\ldots\ldots\ldots.(5)$$

b- PLD with Surface Effect Formula:

The surface effect correction to the PLD starts from the finite depth of the nuclear potential well. The corrections to the full particle-hole PLD are due to eliminated states that have a hole below the bottom of the well. The presence of the nuclear surface region will most strongly affect the initial projectile-target interaction, which produced the composite nucleus. The inclusion of the effect of the nuclear surface imposed by the densities is employed, and simple parameterization of the effective well depth for the first projectile-target interaction can be obtained in calculation of PLD [7-10] as:

$$W(p,h,E,V)=W(p,h,E,\infty)f(p,h,E,V)\ ..(6)$$

where W(p,h,E,∞) is the infinite well resulting from Williams formula and the function (f) represents the surface correction [7] :

$$f(p,h,E,V)=\sum_{i=0}^{h}(-1)^i\binom{h}{i}\left(\frac{E-i\,V}{E}\right)^{n-1}\Theta(E-i\,V)$$

……………………………………….(7)

$\binom{h}{i}$ is the binomial coefficient , V is the central depth of the nuclear potential:

$$V=\begin{cases} V_\circ = 38\,MeV & for \quad h \rangle 1 \\ \hat{V}_1 \quad for & h=1 \end{cases}$$

………………………………………….(8)

, and $\hat{V}_1$ is the effective well depth.

The main surface effect is to reduce the amount of excitation energy which a hole degree of freedom can carry.

c- The PLD with Spin Dependence Formula:

The ESM model was based on a "phase-space" assumption. This means that only the state of energy counts in the development of calculations. Since the angular distribution is very important in nuclear reactions, then one cannot afford to lose this information during model calculations. Therefore, the angular distribution of the emitted particles was added to the model systematically [11]. The particle-hole state densities with spin dependence, W(n, E, J), are assumed to be factorized by a Gaussian distribution function of an angular momentum J [12-14] such that:

$W(n, E, J) = W(n, E)\, R_n(J)$ ....(9)

R_n (J) is angular momentum distribution function which is given as:

$$R_n(J)=\frac{2J+1}{2(2\pi)^{\frac{1}{2}}\sigma_n^3}\exp\left(\frac{-(J+1/2)}{2\sigma_n^2}\right) \ldots\ldots (10)$$

The exciton-dependent spin cut-off parameter (σ_n) can be expressed in different formulas such as[15,16] , respectively :

$\sigma_n^2 = 0.16\, n\, A^{2/3}$ ………….(11)

$\sigma_n^2 = 0.26\, n\, A^{2/3}$

d- PLD with Pairing Effect Formula:

The most important component of the residual nuclear interactions is the pairing force, which is a strong, attractive short-range two-body interaction that couples together pairs of identical nucleons [14]. However, due to the pairing energy of each pair of same kind of particles, one has to add an additional energy to break such pairs for nuclei with even number of nucleons of either type, in addition to the energy needed to excite them. Then, the effective excitation energy is produced from subtracting the pairing energy from excitation energy (incident energy).The Pauli correction was also modified to be consistent with pairing correction, so that the PSD formula becomes [15]:

$$W(p,h,E,P+B_{ph})=\frac{g^n(E+P+B_{ph})^{n-1}}{p!\,h!\,(n-1)!} \ldots$$

…………………………………….(12)

where B_{ph} is the modified Pauli correction following the Williams term $A_{ph.}$,

$$B_{ph}=A_{ph}\sqrt{1+(2g\Delta/n)^2} \quad \ldots\ldots\ldots.(13)$$

and P is the pairing correction term, which is determined by the ground-and excited state gaps $\triangle_o$ and $\triangle$(p,h,E),

$$P=\frac{1}{4}g\,(\Delta_\circ^2-\Delta^2) \quad \ldots\ldots\ldots\ldots\ldots\ldots.(14)$$

$\triangle_o$ can be obtained from the following fitting expression [14] :

$\triangle_o = \triangle_N + \triangle_Z$ ……………………………….(15).

where the neutron and proton gaps are:

$\triangle_N = 1.374 - 0.00516\, N$

………… *(16).*

$\triangle_Z = 1.654 - 0.00958\, Z$

It is related to the condensation energy $C_o = g\triangle_o^2/4$. Then, $\triangle$(p,h,E) can be calculated from the following parameterizations formula [15, 17]:

$$\frac{\Delta}{\Delta_\circ}=\begin{cases} 0.996-1.76(n/n_c)^{1.6}(E/C_\circ)^{-0.68} if & E \geq E_{phase} \\ 0 \qquad\qquad if & E \langle E_{phase} \end{cases}$$

(17)

where $n_c = 0.792\, g\triangle_o$ is the critical number of excitons and E_{phase} is the energy of pairing phase transition given by:

$$E_{phase}=\begin{cases} C_{\circ}\left[0.716+2.44(n/n_{c})^{2.17}\right] & if \quad n/n_{c}\leq 0.446 \\ 0 & if \quad n/n_{c}\rangle\, 0.446 \end{cases}$$

(18)

Actually, the lower limit in eq.(18) was adopted [17] in order to take into account explicitly the lack of a phase transition for small n. The above respective equations provide PLD values even below the minimum excitation energy (threshold: U_{th}) characteristic of each configuration:

$$U_{th}=\begin{cases} C_{\circ}\left[3.23(n/n_{c})-1.57(n/n_{c})^{2}\right] & if \quad n/n_{c}\leq 0.496 \\ C_{\circ}\left[1+0.627(n/n_{c})^{2}\right] & if \quad n/n_{c}\rangle\, 0.446 \end{cases}$$

………………………………….. (19)

III. PLD with Finite Potential Well (non-ESM):

At the increase of excitation energy, g is not being constant anymore, because the spacing between levels (D=1/g) becomes varying with excitation energy [14]. Therefore, the effect of finite depth potential well becomes very important [7, 11, 18]. In fact, every potential well with a finite depth has a finite value of Fermi energy (E_f or ε_o), and its single-particle states have a non equidistant spacing g(ε). The corresponding single particle state density is [12, 19, and 20]:

$$g(\varepsilon)=K_{FG}\varepsilon^{\frac{1}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

where (K_{FG} = $\frac{3}{2}$ A / $\varepsilon_o^{3/2}$) for the Fermi gas square potential well. For the truncated harmonic oscillator there is a different formula. For the square well, the Fermi energy have been assumed as ε_o = 20 MeV , and at this energy g becomes :

$$g(\varepsilon_{\circ})=\frac{A}{13.3} \quad MeV^{-1} \ldots\ldots\ldots\ldots\ldots\ldots(21)$$

IV. The Composite PLD Formulas:

The composite formula includes the previous correction of PLD within the frame work of ESM calculations. In addition to the bound state condition, which is applied to particles, their energy must not exceed the particle binding energy(B), [21, 22], in addition to a back – shift energy (S) due to shell effects [23]. Therefore, the PLD calculations are represented by three composite formulas:

a- The composite formula in the ESM **:** The single level density, which was used to calculate the PLD, is obtained from the ESM [18] as:

$g=g_{p}+g_{h}=\frac{p}{n}g+\frac{h}{n}g$, then the PLD takes the form :

$$W(p,h,E)=\frac{g^{n}E^{n-1}}{p\,!\,h!(n-1)!}f_{k}(p,h,E,F)$$

……………………………………….(22)

where,

$$f_{k}=\sum_{i=0}^{p}\sum_{j=0}^{h}(-1)^{i+j}C_{p}^{i}C_{h}^{j}\left(\frac{E-A_{k}(p,h)-S-iB-jF)^{n-1}}{E}\right) \\ \times\Theta\left(E-E_{thresh}-S-iB-jF\right)$$

…………………………………….(23)

and,

$$A_{k}(p,h)=E_{thresh}(p,h)-\frac{p(p+1)+h(h+1)}{4g}+\frac{(p-1)^{2}(h-1)^{2}}{gF(p,h)}$$

……………………………………..(24)

which is the modified Pauli and paring corrections, also including the effect of passive holes [17], and E_{thresh} is the modified form of the threshold energy for a given exciton configuration with inclusion of the pairing interaction and P_m is maximum(p,h).

$$E_{thresh}(p,h)=\frac{g(\Delta_{\circ}^{2}-\Delta^{2})}{4}+p_{m}\left[\left(\frac{p_{m}}{g}\right)^{2}+\Delta^{2}\right]^{\frac{1}{2}}$$

…………………………………… (25)

and,

$$F(p,h)=12+4g\left[E-E_{thresh}(p,h)\right]/p_{m}$$

…………………………..……..(26)

b-The composite formula with NESM:

In NESM g_p and g_h are varied with energy by taking the Fermi energy as F_o = 38 MeV:

$$W(p,h,E)=\frac{[g_p(p,h)]^p[g_h(p,h)]^h E^{n-1}}{p!h!(n-1)!}f_k(p,h,E,F)$$

……………………………………(27)

where:

$$g_p(p,h)=g_p(\bar{u}_p)=g_\circ\left(\frac{F_\circ+\bar{u}_p}{F_\circ}\right)^{\frac{1}{2}},$$

……………………………………(28)

$$g_h(p,h)=g_h(\bar{u}_h)=g_\circ\left(\frac{F_\circ+\bar{u}_h}{F_\circ}\right)^{\frac{1}{2}}$$

$$\bar{u}_p=\frac{E}{n}\frac{f_k^+(p,h,E,F)}{f_k(p,h,E,F)}\quad,$$

…………..(29)

$$\bar{u}_h=\frac{E-p\,\bar{u}_p}{h}$$

$$f_K^+(p,h,E,F)=\sum_{i=0}^{p}\sum_{j=0}^{h}(-1)^{i+j}C_p^iC_h^i\left(\frac{E-A_k(p,h)-s-iB-jF}{E}\right)^n \times\left(1+\frac{\mathrm{n}}{\mathrm{p}}\frac{iB}{E-A_k(p,h)-s-iB-jF}\right) \Theta(E-E_{thresh}-s-iB-jF)$$

………………………………………(30)

c- The composite formula with NESM and spin distribution

The spin distribution is considered with eq. (27) as follows :

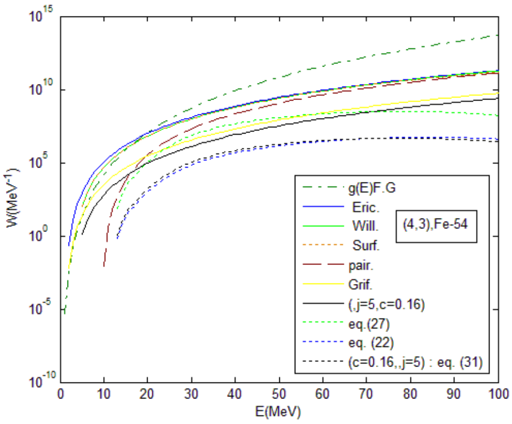
$$W(p,h,E,j)=\frac{[g_p(p,h)]^p[g_h(p,h)]^h E^{n-1}}{p!h!(n-1)!}f_k(p,h,E,F)\,R_n(J)$$

……………………………………(31)

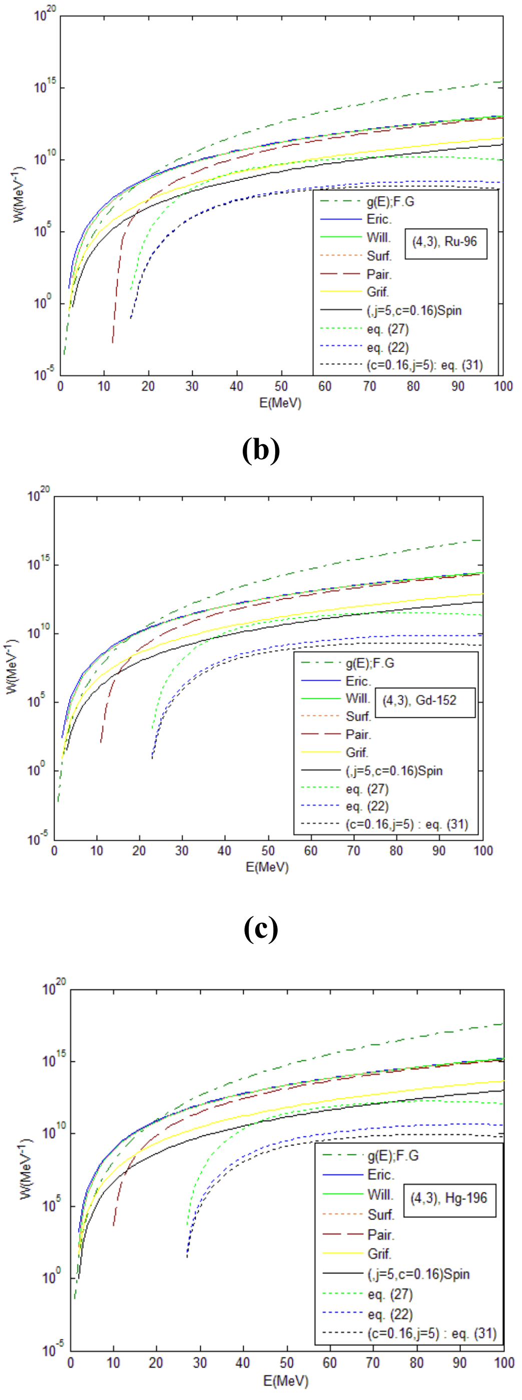
V. Results and Discussion :

The comparison of the calculated PLD as a function of excitation energy, E, based on all the previous formulas, with constant configuration and mass number ^{54}Fe is shown in Fig.(1- a). From this comparison it can be seen that : the calculated PLD which is based on eq.(31) , is less than others, because this formula contains all the corrections. It is so close from the value of PLD based on the composite ESM formula based on Eq.(22). Also, it is obvious that, as more corrections are added, the calculated PLD values decrease especially at higher excitation energies. This is mostly explained due to reduction in the effective excitation energy. However, the effects of some corrections such as Pauli effect, surface effect and spin distribution, decrease the state density values regardless of the excitation energy.

The most approached values of PLD calculated by using Griffin's formula, eq.(1), and William plus spin distribution formula (obtaining the most important corrections as Pauli exclusion principle and spin distribution), and the composite NESM formula (containing all corrections that exceed the PLD and reduce it except for the spin distribution) based on Eq.(27). The PLD is very much affected by the spin distribution when the comparison was made between PLD based on NESM with and without spin distribution. The E_{thresh} of PLD is larger when eq.(31) is used. From the comparison of Figs.(1- a, b, c and d), where Fig.(b, c, and d) are the same as (a) but with mass number ^{96}Ru, ^{152}Gd and ^{196}Hg , respectively, one can noticed that as A increased , E_{thresh} increases and the difference between the ESM and NESM increases too. Fig.(2)shows the effect of spin distribution as a function of E and A.



(a)



(b)

(c)

(d)

Fig.(1):The comparison of the calculated PLD as a function of E based on all formulas representing all the various corrections, with constant configuration and different mass number (a) for ^{54}Fe, (b) ^{96}Ru , (c) ^{152}Gd , and (d) ^{196}Hg, respectively .

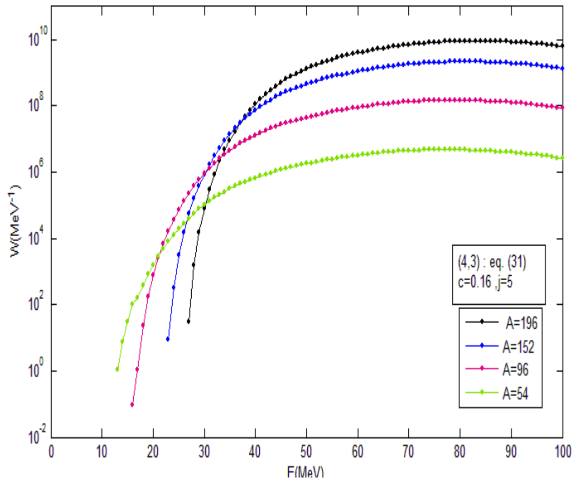
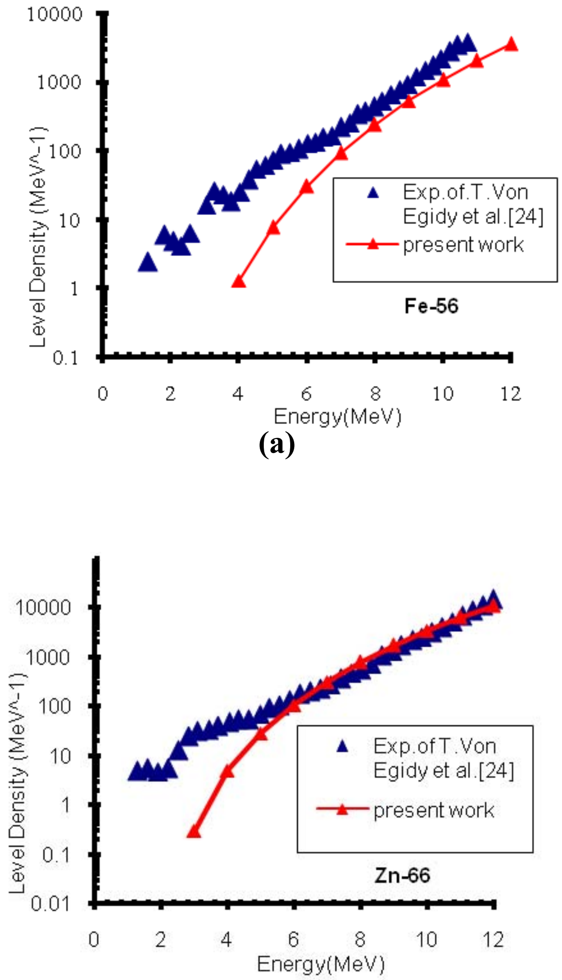


Fig. (2): The comparison of PLD based on NESM formula with spin distribution as a function of E and A, with constant configuration and spin.

Figs.(3-a,b) represent the comparison of the calculated NLD (total level density) with the experimental data for ^{56}Fe and ^{66}Zn [24] respectively. The PLD is calculated by using William formula with spin distribution where the summation is done for n= 1 – ñ. Results of The William's formula with spin distribution is the nearest approach to that of composite formula, that is obtained from comparison result of calculated PLD for nuclei with mass number range 50- 200 and attributed to William's formula with spin distribution obtaining the most important corrections. This evidently proves that a good agreement between our results and other [24].

However, the composite formula is not used because of the effect of the step function that makes E_{thresh} high. In other word, the composite formula can be used for intermediate energy which is approximately ≥ 10 MeV.



(a)

(b)

Fig.(3): Comparison of results of the calculated NLD based on Williams' formula plus spin distribution, where J= 5 and spin cut off parameter obtain the formula of Herman and Reffo [16], with experimental data for (a) ^{56}Fe and (b) ^{66}Zn

VI. Conclusions:

From the above results, one can be conclude that the Pauli term reduces the PLD values because of its nature which blocks the states. The effect of spin distribution function increases with increasing E and A. Also, it is reduces the PLD values, so that the PLD increases smoothly with increasing E and A. The PLD values are reduced by the pairing effect at low incident energy. The pairing effect increases with decreasing E. The effect of the finite depth on PLD disappeares when the incident energy is equal to the Fermi energy (E_f) of the potential well. This effect reduces the PLD when E < E_f. As more corrections are added, the calculated PLD values decreased especially at higher excitation energies. Also, it was found that William's plus spin formula, which has been inserted in the PLD formula is the most successful approach to the composite formula and agrees with standard experimental results.

References :

[1] J. R. Huizenga, and L. G. Meretto, Annu. Rev. Nucl. Sci. 22(**1972**) 427-464.

[2] H. A. Bethe, Rev. Mod. Phys. 9(**1937**)69.

[3] H. A. Bethe, Phys. Rev. 50(**1936**)332.

[4] J.J. Griffin, Phys. Rev. Lett. 17 (**1966**) 478.

[5] T. Ericson, Adv. Phys. 9 (**1960**) 425.

[6] F.C. Williams, Nucl. Phys. A 166 (**1971**) 231.

[7] C. Kalbach, Phys. Rev. C32 (**1985**) 1157.

[8] M. Avrigeanu, A. Harangozo and V. Avrigeanu, Phys. Rev. C54 (**1996**) 2538.

[9] C. Kalbach, Phys. Rev. C69 (**2004**) 014605-1.

[10] C. Kalbach, Phys. Rev. C73 (**2006**) 024614-1.

[11] C. Kalbach, Phys. Rev. C23 (**1981**) 124.

[12] E. Betak and B. E. Hodgson, Rep. Prog. Phys.61 (**1998**) 483, OUNP-98-02 University of Oxford.

[13] K. Van Houcke, S. M. A. Rombouts, K. Heyde and Y. Alhassid, Archive of Cornell Univ. Library, Ref: nucl-th / 0702041(**2007**).

[14] P. E. Hodgson, " Nuclear Reactions and Nuclear Structure", Clarendon Press. OXFORD, **1971**, P(308).

[15] C. Y. Fu, Nucl. Sci. Eng., 92 (**1986**) 440.

[16] M. Herman and G. Reffo, Phys. Rev. C36 (**1987**) 1546.

[17] C. Kalbach, Nucl. Sci. Eng. 95 (**1987**) 70.

[18] M. Arrigeanu and V. Arrigeanu, Archive of Cornell Univ. Library, Ref: physics/ 9805002 V1 1 may **1998**.

[19] E. Gadioli, E. Gadioli ErBa and P.G. Sona, Nucl. Phys. A217 (**1973**) 589.

[20] C. Bloch, Phys. Rev. 93(**1954**)1094.

[21] C. Kalbach, Phys. Rev. C24 (**1981**) 819.

[22] K. Stankiewicz, A. Marcinkowski and M. Herman, Nucl. Phys. A435 (**1985**) 67.

[23] W. Dilg, W. Schantl, H. Vonach and M. Uhl, Nucl. Phys. A217 (**1973**) 269.

[24] T. Von Egidy, D. Bucurescu, Phys. Rev. C72(**2005**)