Particle-Hole State Density Calculations with Non-Equidistant Spacing Model: I. Basic Derivation

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Abstract

The basic analytical formula for particle-hole state densities is derived based on the non-Equidistant Spacing Model (non-ESM) for the single-particle level density (s.p.l.d.) dependence on particle excitation energy *u*. Two methods are illustrated in this work, the first depends on Taylor series expansion of the s.p.l.d. about *u*, while the second uses direct analytical derivation of the state density formula. This treatment is applied for a system composing from one kind of fermions and for uncorrected physical system. The important corrections due to Pauli blocking was added to the present formula. Analytical comparisons with the standard formulae for ESM are made and it is shown that the solution reduces to earlier formulae providing more general way to calculate state density. Numerical calculations then are made and the results show that state density behavior with excitation energy deviates from Ericson's and Williams' formulae types, especially at higher excitation energies.

Keywords
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حسابات كثافة الحالة لجسيمة-فجوة حسب أنموذج التباعد غير المتساوي: أ. الصياغة الأساسية

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الخلاصة

أُشتقتْ الصيغة التحليلية الأساسية الخاصة بكثافة الحالات الخاصة بنظام جسيمة-فجوة استنادا على أنموذج التباعد الحقيقي غير المتساوي (non-ESM) المعتمد على علاقة كثافة مستوى الحالة لجسيمة مفردة (s.p.l.d.) مع طاقة تهيج الجسيمة *u*. أُتبعتْ طريقتان في هذا البحث، الطريقة الأولى تعتمد على مفكوكة سلسلة تايلور للـs.p.l.d. حول *u*, بينما كانت الطريقة الثانية معتمدة على الاشتقاق المباشر لمعادلة كثافة الحالات. ثم طُبقتْ هذه المعالجة على نظام مؤلف من نوع واحد من الجسيمات ولنظام فيزياوي غير مصحح وأُدخلتْ التصحيحات لطاقة باولي بطريقة مباشرة إلى المعادلة الحالية. من المقارنات التحليلية التي أُجريتْ مع معادلات سابقة خاصة بالنظام المتساوي البعد (ESM) تم التوصل إلى أن المعادلة الحالية التي تم اشتقاقها تعبر عن حالة عامة لحساب كثافة الحالات. أما المقارنات العددية التي أجريت فقد أوضحت بأن النتائج الحالية تنحرف عن صيغتي أيركسون ووليماز،خاصة عند الطاقات العالية.

Introduction

The exciton model [1] provides a good approach to describe continuum emission, where the intermediate stages are treated semi-classically to explain the preequilibrium emission (PE) of the excited nuclei. An important physical quantity needed in this course is the particle-hole state density which describes

the population of the single-particle states per energy interval.

The single particle level density (s.p.l.d.), is used to describe the dependence of the state density, $\omega(p,h,E)$, on the particle number p, hole number h, and excitation energy E. The exciton number, n, is given by the sum $(p+h)$. Assuming p-particles above Fermi surface, and h-holes below it, then the exciton number represents the principal degree of freedom that the state density depends on. If one assumes that the s.p.l.d. is energy-independent, *i.e.,* g is constant, then the model is called the Equidistant Spacing Model, (ESM). The s.p.l.d. in this case is usually approximated by the relation $g \sim A/d$, where A is the mass number of the nucleus and d is the energy spacing and the values used for this parameter varies from 8 to 26 MeV, depending on the mass of the nucleus. The ESM approach was used to derive the uncorrected state density formula usually known as Ericson's formula [2].

$$\omega^{E}(p,h,E)=\frac{g^{n}E^{n-1}}{p!h!(n-1)!} \qquad (1),$$

and to obtain the formula that includes Pauli blocking energy in the energy part, also known as Williams' formula [3],

$$\omega^{W}(p,h,E)=\frac{g^{n}(E-A_{p,h})^{n-1}}{p!h!(n-1)!} \qquad (2),$$

where $A_{p,h}$ is Pauli blocking defined by,

$$A_{p,h}=\frac{p^{2}+h^{2}+p-h}{4g}-\frac{h}{2g} \qquad (3).$$

Eqs.(1-3) are used for state density calculations assuming a system of non-interacting particles of the same kind, *i.e.*, one-component Fermi gas, FG.

There have been many developments to these equations in order to add important corrections, such as pairing [4-6], shell structure including finite well depth correction and surface effect [7-9], charge effect [10], linear and angular momentum distribution [11-14] and isospin [15-18]. Even if one includes all the above corrections during state density calculations, the obtained values still show some inconsistency with experimental results, specially during analyzing spectra with high and low excitation energies for mass numbers less than 15 or larger than 150 [10]. This put the ESM approach under increasing debate [19-20], and it revealed the necessity to find other more accurate approaches to describe the state density. However, the ESM is treated as an approximation in most cases, because the actual nuclear states depend on the excitation energy and the exciton energy u as well, *i.e.*, $g=g(u)$. Therefore, some attempts to replace the ESM approach were made only few years after the proposal of the exciton model. For example, Williams [21], Albrecht and Blann [22], and Herman et al. [23-24] used numerical methods to calculate the level and state density. Other methods such as Shell Model approximate calculations [25], Shell Model Monte Carlo (SMMC) [26-27], and projected SMMC [28-30] were focused on state density calculations. In addition, the state and level density for deformed nuclei was a subject of interest to many studies [31-32].

The need for simple analytical description of the nuclear state density away from the ESM approach is still evident [20]. Bogila et al. [33] proposed an approximate method for particle-hole state density calculations for non-ESM by expanding the s.p.l.d. dependence on the excitation energy via Taylor series expansion and taking only the first three terms. Harangozo et al. [20] extended further this method to include the effects of the nucleon binding energy, B, and Fermi energy of the system in a more reliable way.

An exact analytical method is given in this paper to calculate nuclear state density for any given exciton configuration away from the ESM

approach. The present method is basically an extension of the work of Bogila et al. [33] and Harangozo et al. [20] but without any further approximations. The exact form of the state density calculation for non-ESM dependence of the s.p.l.d., is given, where we try to extend the methods mentioned above. However, the excitation energy dependence given here does not explicitly include the various effects mentioned at the beginning, but they are added to the main equation by inspection.

Particle-Hole State Density with non-ESM

A. The s.p.l.d.

The s.p.l.d. is the key at which the expression of the state density depends on. Earlier attempts were to consider variable Fermi level F [34] or more free energy dependence on u below and above Fermi surface [35, 36]. Kalbach [9] discussed this dependency in some details and the conclusion was made is that, regardless the specified type of the potential well, the s.p.l.d. is expected to vary between that of the simple square well potential to the simple harmonic oscillator. In all cases, the required task is to find the proper dependence of the s.p.l.d. on energy. Below is a description of the method given by Bogila et al. [33] and Harangozo et al. [20] to illustrate such dependence. In Table (1) a list of the symbols used in this work is given.

Consider a system of indistinguishable types of particles p and their holes h. In the frame of FG model, the s.p.l.d. is given as,

$$g(\varepsilon)=g_o\sqrt{\frac{\varepsilon}{F}} \qquad (4),$$

where $g_o=3A/2F$, is the s.p.l.d. at Fermi level energy. F is defined for the nuclear system under study by the relation, $F=\left(\frac{9\pi}{8}\right)^{2/3}\frac{\hbar^2}{2mr_o^2}$, where r_o~1.2 to 1.5 fm, depending on the size of the nucleus, and for particles and holes we have

$$\left.\begin{aligned}\varepsilon&=F+u_p\\ \varepsilon&=F-u_h\end{aligned}\right\} \qquad (5).$$

Thus, we can write the following,

$$\left.\begin{aligned}g_p(u_p)&=g_o\sqrt{1+\frac{u}{F}}\\ g_h(u_h)&=g_o\sqrt{1-\frac{u}{F}}\end{aligned}\right\} \qquad (6).$$

In order to have well-behaved s.p.l.d. dependence on u, eq.(6) must be governed by suitable Heaviside function so that at the un-allowed limit $u^{(h)}>F$ the results still converge. Thus,

$$\left.\begin{aligned}g_p(u)&=\sqrt{1+\frac{u}{F}}\;\Theta(B-u)\\ g_h(u)&=\sqrt{1-\frac{u}{F}}\;\Theta(F-u)\end{aligned}\right\} \qquad (7).$$

The state density then can be found from the relation [20],

$$\omega(p,h,E)=\frac{1}{p!h!}\int_0^\infty du_1^{(p)}g_p(u_1^{(p)})\int_0^\infty du_2^{(p)}g_p(u_2^{(p)})..\int_0^\infty du_p^{(p)}g_p(u_p^{(p)})\times$$
$$\int_0^\infty du_1^{(h)}g_h(u_1^{(h)})\int_0^\infty du_2^{(h)}g_h(u_2^{(h)})..\int_0^\infty du_h^{(h)}g_h(u_h^{(h)})\delta\left(E-\sum_{\lambda=1}^{p}u_\lambda^{(p)}-\sum_{j=1}^{h}u_j^{(h)}\right) \qquad (8),$$

where Dirac delta function, δ in eq.(8), can be given in its integral form as,

$$\delta\left(E-\sum_{\lambda=1}^{p}u_\lambda-\sum_{j=1}^{h}u_j\right)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}dk\exp\left[ik\left(E-\sum_{\lambda=1}^{p}u_\lambda-\sum_{j=1}^{h}u_j\right)\right] \qquad (9).$$

If one uses this equation, then the state density can be given as,

$$\omega(p,h,E) = \frac{1}{2\pi\, p!h!}\int_{-\infty}^{+\infty} \exp(ikE)dk\left(\int_0^{\infty} g_p(u)\exp(-ik\,u)du\right)^p \left(\int_0^{\infty} g_h(u)\exp(-ik\,u)du\right)^h \quad (10).$$

which represents a convolution of the s.p.l.d. for particles and holes with Dirac function.

Table(1). List of symbols used in this work.

A	Mass number of the target nucleus
$A_{p,h}$	Pauli blocking energy
B	Binding energy of the emitted nucleon
$B_{p,h}$	Modified Pauli energy
C^p and C^h	Coefficients of integrating $P(k)$ and $H(k)$
C_e	Condensation energy
D^p and D^h	Coefficients of expanding eq.(6)
E	Excitation energy
E_{phase}	Pairing energy due to phase transition
F	Fermi energy of the target nucleus
$P(k)$ and $H(k)$	Functions that represent part of the state density integration
$P(\Delta)$	Pairing energy
$\Theta(x\text{-}x_o)$	The Heaviside step function
d	Energy spacing in the ESM approach
g_o ,g	Single-particle level density (s.p.l.d.) for ground and excited states
Δ_o, Δ	Energy gap of the nuclear levels for the ground and excited states
ε	Exciton energy
$\omega(p, h, E)$	State density of the system for p particles, h holes and excitation energy E
$u^{(p)}$ and $u^{(h)}$	Single particle and hole excitation energies
n, p, h	Exciton, particle and hole numbers; $n=p+h$
n_c	Critical exciton number
r_o, m	Nucleon classical radius and its effective rest mass

B. State density formula

The difference between the present paper and the method used in [20] and [33] is that, in here, the entire expansion of eq.(6) is taken, and not only the first three terms. For simplicity, further assumptions (such as the effects of B and Heaviside step function) are omitted for the time being. These are considered as modifications and will be added later. Using Maclaurin expansion of eq.(6) about u and

rearranging the terms, then **the first method** is to write,

$$g_p(u) = g_o\sum_{m=0}^{\infty} \frac{D_m^p}{m!}\left(\frac{u}{F}\right)^m \quad (11),$$

$$g_h(u) = g_o\sum_{m=0}^{\infty} \frac{D_m^h}{m!}\left(\frac{u}{F}\right)^m \quad (12),$$

where, for later convenience, the defined coefficients are given in such away that,

$$\left.\begin{aligned} D_m^p &= \begin{cases} 1 & m=0, \\ (-1)^{m+1}\dfrac{(|2m-3|)!!}{2^m} & m\geq 1, \end{cases} \\ D_m^h &= \begin{cases} 1 & m=0, \\ -\dfrac{(|2m-3|)!!}{2^m} & m\geq 1, \end{cases} \end{aligned}\right\} \quad (13),$$

which reduces to the forms given by refs.[20, 33] if one takes the first three terms only. Let us first re-write eq.(10) in a simpler and more direct form as,

$$\omega(p,h,E)=\frac{1}{2\pi\, p!h!}\int_{-\infty}^{+\infty}\exp(ikE)\left[P(k)\right]^{p}\left[H(k)\right]^{h}\,dk \qquad (14).$$

Two methods are applicable at this point, one can either use the expansions (11) and (12) to find the solution of eq.(14), which will give a solution that depends on how many terms we take from Taylor-Maclaurin expansion. Inclusion of the entire terms will give a mathematically exact solution.

The second method, which we shall follow in the remaining of this paper, is to substitute eq.(6) directly as it is into eq.(14) and solve the integral

$$P(k)=\int_{0}^{\infty} g_p(u)\exp(-i\,k\,u)\,du = g_o\, F\int_{0}^{\infty}\exp(-i\,k\,u)\sqrt{1+\frac{u}{F}}\,du \qquad (15),$$

and a similar expression for $H(k)$. The problem solves to, after some algebra,

$$\therefore P(k)=\frac{g_o\,F}{2\sqrt{\pi}}\sum_{m=0}^{\infty}\frac{C_m^{p}}{(iFk)^{m+1}} \qquad (16),$$

$$H(k)=\frac{g_o\,F}{2\sqrt{\pi}}\sum_{m=0}^{\infty}\frac{C_m^{h}}{(iFk)^{m+1}} \qquad (17),$$

where the coefficients here are redefined in a different way than eq.(13), as,

$$\left.\begin{aligned} C_m^{p} &= (-1)^{m+1}\left(m-\frac{3}{2}\right)! \\ C_m^{h} &= -\left(m-\frac{3}{2}\right)! \end{aligned}\right\} \qquad (18).$$

Grouping these with eq.(14) and arranging terms of the solution, one reaches to the formula,

$$\omega(p,h,E)=\frac{1}{p!h!}\frac{g_o^{n}\,F^{n}}{2^{n+1}\pi^{(n+1)/2}}\sum_{\substack{a_1,a_2..\\ a_p=0}}^{\infty}\prod_{j=1}^{p}C_{a_j}^{p}\sum_{\substack{b_1,b_2..\\ b_h=0}}^{\infty}\prod_{\lambda=1}^{h}C_{b_\lambda}^{h}\int_{-\infty}^{+\infty}\frac{\exp[i\,k\,E]dk}{(iFk)^{n+a_1+a_2+...+a_p+b_1+b_2+...+b_h}} \qquad (19).$$

On further making the following definition,

$$N=n+\sum_{j=1}^{p}a_j+\sum_{\lambda=1}^{h}b_\lambda$$

and solving using Cauchy's integral formula, one can obtain the following solution,

$$\therefore \omega(p,h,E)=\frac{g_o^{n}}{2^{n}\pi^{n/2}\,p!h!}\widehat{\Xi}\,\frac{E^{N-1}}{F^{N-n}(N-1)!} \qquad (21),$$

where, for the sake of saving space and simplicity, we have make the name of the following special mathematical multiplication operator $\widehat{\Xi}$, as,

$$\widehat{\Xi}\equiv\sum_{\substack{a_1,a_2,..a_p,\\ b_1,b_2,..b_h=0}}^{\infty}\prod_{j=1}^{p}C_{a_j}^{p}\prod_{\lambda=1}^{h}C_{b_\lambda}^{h} \qquad (22).$$

Eq.(22) justifies the proper definition of the coefficients in eq.(18).

Eq.(21) actually shows that including eq.(6) without expansion will lead to the addition of more terms to Ericson's formula, eq.(1).The higher terms are associated with the dependence of $1/F^{N-n}$ which forces the higher terms to converge rapidly as N (and hence, the expansion indices a's and b's) increase.

C. Analytical Comparisons

1. With Ericson's Formula

It is convenient in the present point to check whether the solution above, eq.(21), reduces to Ericson's formula, eq.(1), at the limits $a_1=a_2=..=b_1=b_2=..=0$. These limits mean that we took g as being a constant of energy. Indeed, at these limits, then one will have $N=n$ and the mathematical operator $\widehat{\Xi}$ reduces to $(2^{n}\,\pi^{n/2})$. Clearly this will make eq.(21) to be equal to eq.(1), Ericson's formula.

2. With Bogila et al.

The equation due to Bogila et al. [33] is,

$$\therefore \omega(p,h,E) = \omega^{\mathrm{E}}(p,h,E)\sum_{k_1}^{p}\sum_{i_1}^{k_1}\sum_{l_1}^{h}\sum_{j_1}^{l_1} (-1)^{j_1+l_1} C_p^{k_1} C_{k_1}^{i_1} C_h^{l_1} C_{l_1}^{j_1} \times \left(\frac{g_o^{''}}{g_o^{'}}\right)^{i_1+j_1}\left(\frac{g_o^{'}}{g_o}\right)^{k_1+l_1} \frac{E^{i_1+j_1+k_1+l_1}(n-1)!}{(n-1+i_1+j_1+k_1+l_1)!} \quad (23),$$

where $g_o^{''} = -\frac{g_o}{8F^2}$, $g_o^{'} = \frac{1}{2F}$, if one rewrites it as, using eq.(1), then letting $k_1=a$, $i_1=s$, $l_1=b$, $j_1=q$, and defining N for this case is their sum, so we can write the compact form,

$$\therefore \omega(p,h,E) = \frac{g_o^n}{p!\,h!}\sum_{a}^{p}\sum_{s}^{a}\sum_{b}^{h} \sum_{q}^{b}(-1)^{b+q+a+s}\prod_{j=1}^{2}C_p^j\prod_{i=1}^{2}C_h^i \times \frac{1}{2^{2s+2q+a+b}F^{N-n}}\frac{E^{N-1}}{(N-1)!} \quad (24)$$

which, on comparing with eq.(21), one can see the close equality if one sets the indices to change from zero to 2, *i.e.,* when taking the first three terms only. In eq.(24), the difference in the summations limits are ordered this way because the expansion was limited to take only few terms, thus if one wants to raise the resulting expansion sums to the powers of p and h -in order to apply eq.(14)- one usually uses the binomial method. However, since in the present method extends to take eq.(6) as it is without expansion, other methods were used here to ensure convergence of these summations.

Results and Discussions

In order to check the accuracy of the present treatment, we choose to compare the numerical results with more than one standard formula of the state density. We thus aim to test this procedure regardless these restrictions, but we chose F to have two values: 38 MeV and 100 MeV and perform calculation that are follow the condition $E<F$. However, the proper selection of F represents a serious challenge in PE calculations. If one uses the analytical value of F it will be no more that ~ 21.5 MeV in this case, resulting a proper (analytical) value of g_o $=3A/2F=3.907$ MeV^{-1}. On the other hand, setting such small value of F will highly reduce the results at $E > 30$ MeV.

In the present work, the value of g_o was set to be 4 MeV^{-1} by selecting ^{56}Fe nucleus with d equals to 14 MeV. g_o was set to the value given above regardless the chosen Fermi energy. We compare first with Ericson's formula, then with Williams' formula that considers Pauli blocking only, and then with Williams' formula that includes pairing. Finally we compare with realistic 1p-1h state density results.

The results of applying eq.(21) for particle-hole state density calculation are shown below for exciton configurations (1p,1h), (2p,1h) and (3p,2h); which represent the most important configurations in PE calculations. The summation terms in eq.(21) were treated such that the maximum was set to a certain selected value, not to exceed 30, rather than infinity. This is because of the programming limitations. Calculations were made using Matlab code written for this purpose.

A. With Ericson's Formula

In Fig.(1), the results of configurations (1p,1h) - or simply (1,1)- are shown for non-interacting system. First, in the case (*a*) F=38 MeV, only three terms are adequate for practical calculations, where as the number of summation terms increases, the ratio tend to fall. However, using a conveniently large F value, as in the case of Fig.(1-*b*), then the system of calculation will be slightly affected when changing the terms from 3 to 30. The number of terms was not chosen to be the same in both cases because in case (*b*) the differences between 3 and 6 summation terms are insignificant in such away that the two curves can not be distinguished. This dependence on F strongly suggests that the value of F plays the important rule

in deriving and applying eq.(21). Actually, this comes from the basic assumption made at the beginning, that the application of eq.(6) is valid for all values of u, whereas it must be applied for the values satisfying the condition u/F <1; or to use eq.(7) instead.

Second, the general behavior of both cases shows that, as the excitation energy increases in this configuration, the ratio between $\omega^{(\text{non-ESM})}(p,h,E)/\omega^{E}(p,h,E)$ decreases which indicates the importance of corrections at higher energies due to higher terms. This behavior is the same as found by Bogila et al. and Harangozo et al. [20]. It was pointed out before [3] that Ericson's formula overestimates the state density calculated values by a considerable amount at higher energies. At other exciton configurations, the difference will be even higher at low energies as seen from the Figs.(2) and (3) below.

B. With Williams' Formula

Inclusion of Pauli blocking energy is made here by inspection, that is, to add $A(p,h)$ into eq.(21) directly by assuming that the excitation energy E will be reduced by this amount. Then, eq.(21) is rewritten as,

$$\therefore \omega^{non-ESM}(p,h,E) = \frac{g_o^n}{2^n \pi^{n/2}\, p!h!} \widehat{\Xi} \frac{\left(E - A(p,h)\right)^{N-1}}{F^{N-n}(N-1)!} \qquad (25),$$

The results are shown in Figs.(4-6), where in this case the ratio $\omega^{(\text{non-ESM})}(p,h,E)/\omega^{W}(p,h,E)$ is shown as a function of the excitation energy, E. In Figs. (5) and (6), exceptional differences occur for large values of F where the ratio never reaches unity. However, in the case for F=38 MeV, the behavior of three summation terms is consistent with the expected results, and again as the number of terms increase, the ratio tends to drop. It should be mentioned that three terms and thirty terms both result almost the same in Figs.(6-*a* and 6-*b*), where the deviation is less than 2% at maximum energy. Therefore, only results for 30 number of terms are shown. Also note that at these figures, the curves starting from values larger than two, indicating that at such low energies the differences between Williams' formula and eq.(25) are quiet large. This also indicates that inclusion of Pauli blocking energy should be accompanied with more accurate terms.

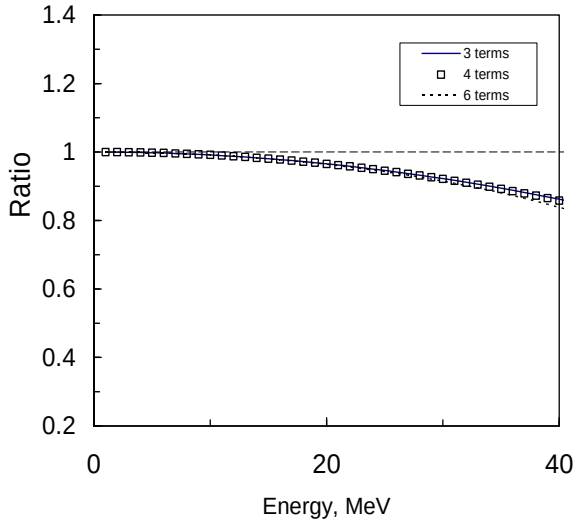

Fig.(1-a). *F*=38 MeV

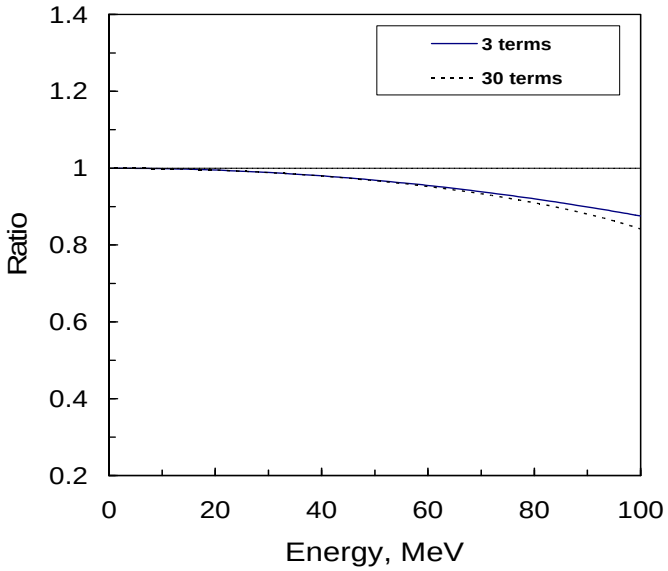

Fig.(1-b). *F*=100 MeV

Fig.(1): The ratio between the results of eq.(21) and Ericson's formula, eq.(1), for (a) F=38 MeV, and (b) F=100 MeV, for configuration (1,1). Target nucleus is ^{56}Fe. In both cases, g_o was set to be 4 MeV^{-1}. The number of summation terms are shown for each case, which represent the maximum number of iterations at which the calculation program terminates

.

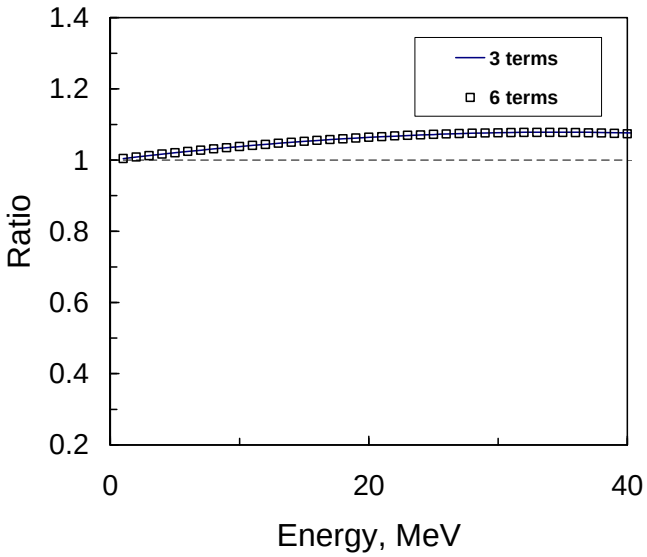

Fig.(2-a). *F*=38 MeV

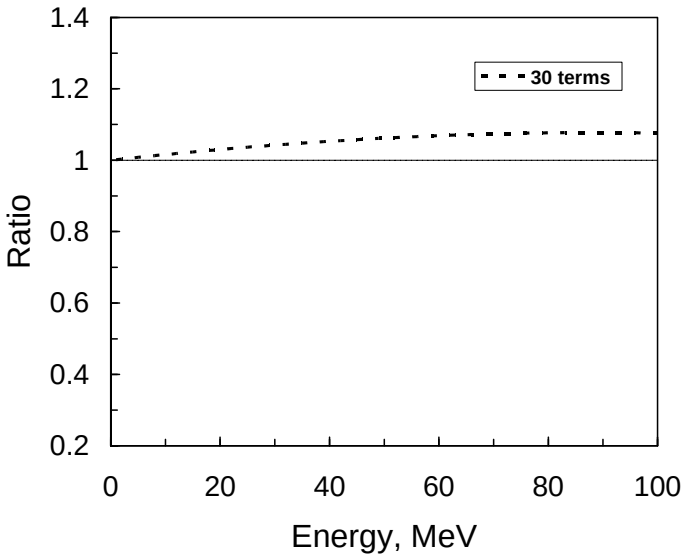

Fig.(2-b) . *F*=100 MeV

Fig.(2): The same as Fig.(1) but for configuration (2,1).

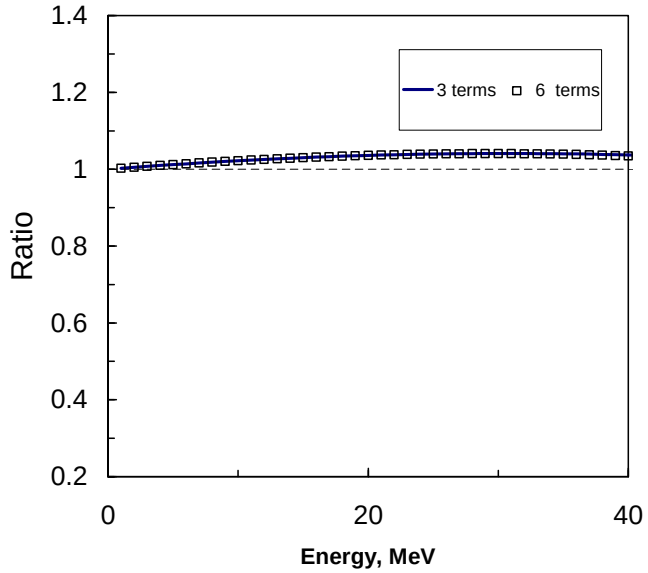

Fig.(3-a). *F*=38 MeV

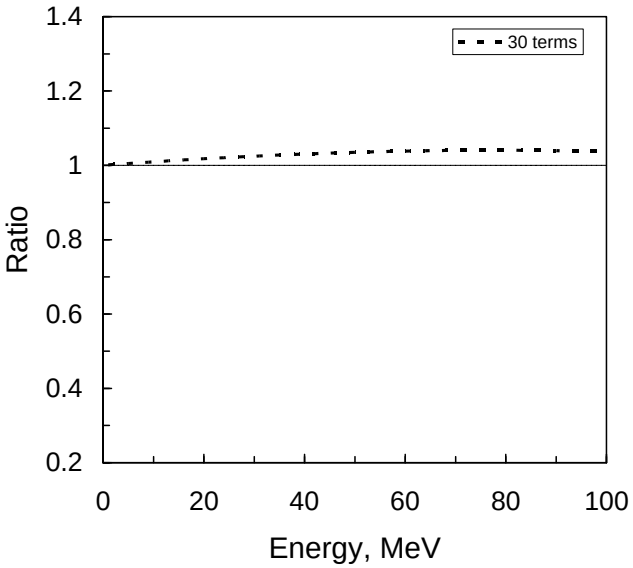

Fig.(3-b). *F*=100 MeV

Fig.(3): The same as Fig.(1) but for configuration (3,2).

Conclusions

In conclusion, it is found that first and most simple solution eq.(21) actually represents the version of Ericson's formula for non-ESM. The s.p.l.d. was used in order to find the analytical expression of the state density. The present treatment shown in eq.(21) is somewhat useful in practical calculations. This formula was put under several numerical comparisons with the standard Ericson's formula, and Williams' formulae that include pairing, both with ESM approach and for the exciton configurations (1,1), (2,1), and (3,2), and it was shown that,

1- Theoretical and numerical comparisons with Ericson's formula for the first term of the solution showed that the present method is in a good consistency. The effects of large Fermi energy were also shown to change the behavior of the state density.

2- Theoretical comparisons with the formulae of Bogila et al. [33] and Harangozo et al. [20] for the first three terms showed that the solution for the present method is actually a general solution that turns into the special cases mentioned above if one takes only three terms of the s.p.l.d. expansion on *u*.

3- Pauli blocking energy was included in eq.(25) and numerical calculations show that the behavior of eq.(21) is slightly improved. This suggested further development of the present treatment. Comparisons with Williams formula indicated better consistency at higher energies, but the results seem to deviate at low energies.

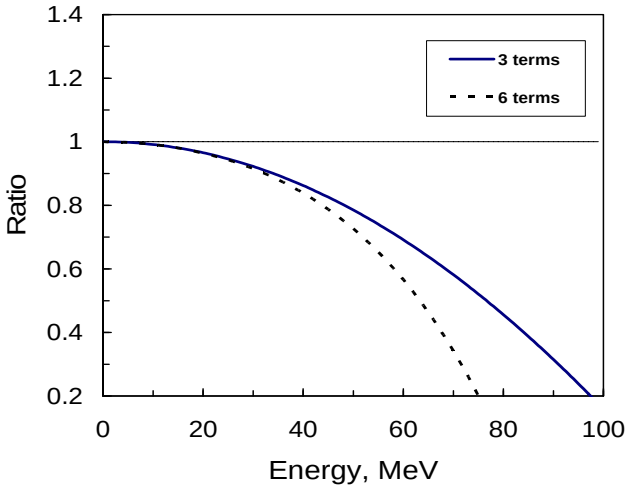

Fig.(4-a). *F*=38 MeV

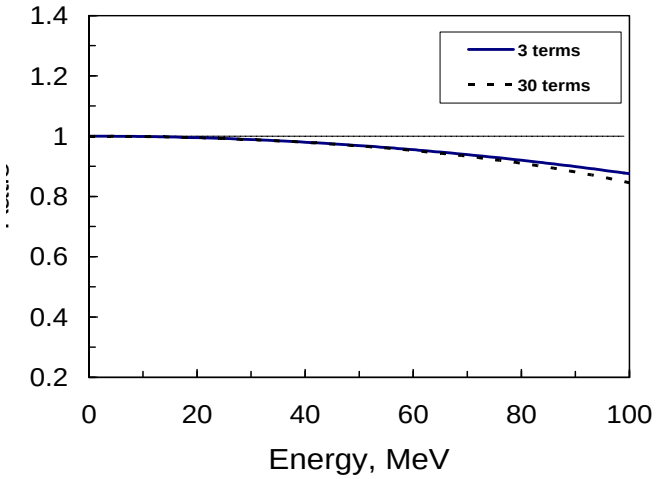

Fig.(4-b). *F*=100 MeV

Fig.(4): The ratio between the results of eq.(25) and Williams' formula, eq.(2), for configuration (1,1).

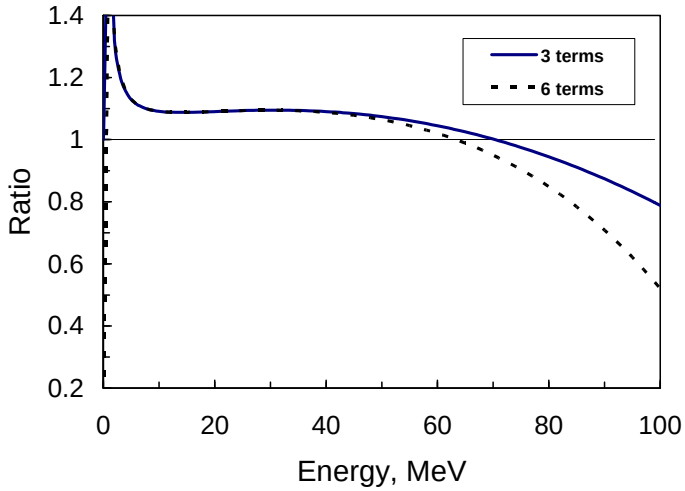

Fig.(5-a). *F*=38 MeV

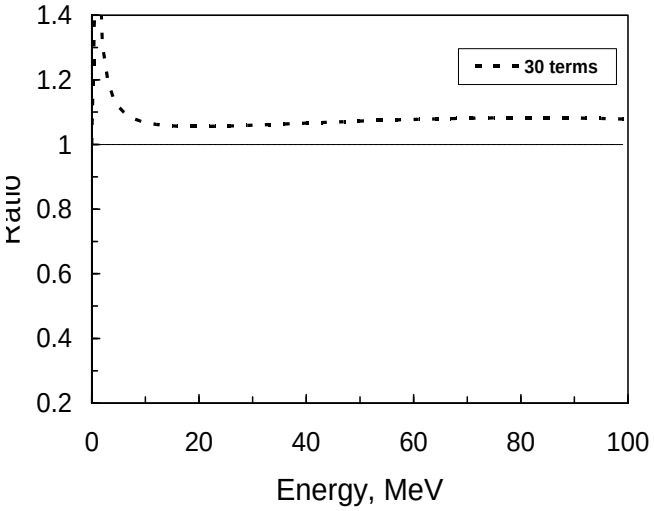

Fig.(5-b). *F*=100 MeV

Fig.(5): The same as Fig.(4) but for configuration (2,1).

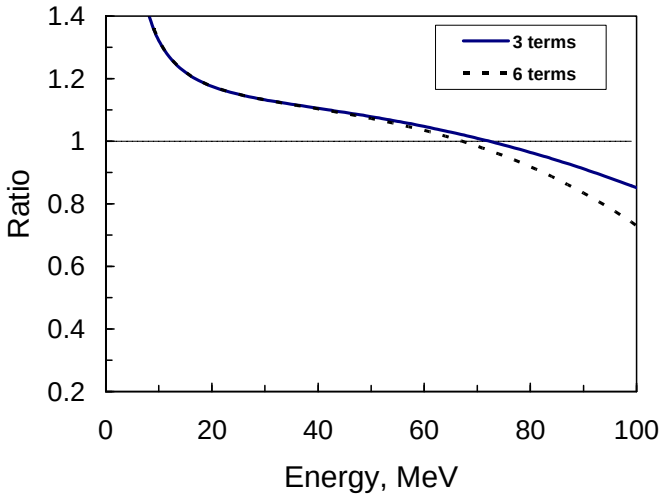

Fig.(6-a). *F*=38 MeV

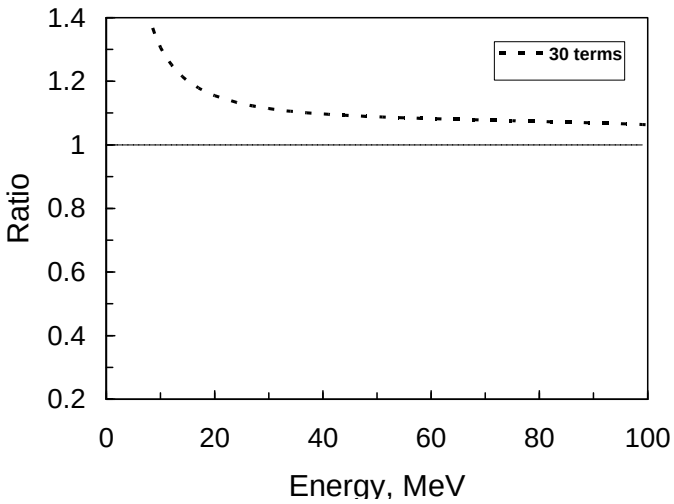

Fig.(6-b). *F*=100 MeV

Fig.(6): The same as Fig.(4) but for configuration (3,2).

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