

The study of nuclear structure for some nuclei

Zahraa A. Abdul Muhsin

Department of Biology, College of Science University of Baghdad, Baghdad, Iraq

E-mail: mohammed8084@yahoo.com

Abstract

An analytical form of the ground state charge density distributions for the low mass fp shell nuclei ($40 \leq A \leq 56$) is derived from a simple method based on the use of the single particle wave functions of the harmonic oscillator potential and the occupation numbers of the states, which are determined from the comparison between theory and experiment.

For investigating the inelastic longitudinal electron scattering form factors, an expression for the transition charge density is studied where the deformation in nuclear collective modes is taken into consideration besides the shell model space transition density. The core polarization transition density is evaluated by adopting the shape of Tassie model together with the derived form of the ground state charge density distribution. In this work, we devote our investigation on $0_1^+3 \rightarrow 2_1^+3$ transition of ^{50}Ti , $0_1^+1 \rightarrow 2_1^+1$ transition of ^{50}Cr and $0_1^+2 \rightarrow 2_1^+2$ of ^{52}Cr nuclei. It is found that the core polarization effects, which represent the collective modes, are essential for reproducing a remarkable agreement between the calculated inelastic longitudinal $C2$ form factors and those of experimental data.

Key words

Charge density,
Harmonic oscillator,
form factor and
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دراسة التركيب النووي لبعض العناصر

زهراء علي عبد المحسن

قسم علوم الحياة، كلية العلوم، جامعة بغداد، بغداد، العراق

الخلاصة

لقد تم اشتقاق صيغة رياضية بسيطة نحسب منها الحالة الأرضية لتوزيعات كثافة الشحنة النووية للمنطقة الواقعة في اسفل القشرة النووية fp (العدد الكتلي لها $40 \leq A \leq 56$). تستند الصيغة الرياضية المشتقة على الدوال الموجية للمتذبذب التوافقي البسيط وعلى أعداد الأشغال للحالات النووية المحسوبة من المقارنة مع النتائج العملية.

درست عوامل التشكل الطولية للأستطارة الألكترونية غير المرنة والمتضمنة انتقال كثافة الشحنة عندما نأخذ بنظر الاعتبار التشوه في الأنماط التجميعية النووية الى جانب كثافة الانتقال لأنموذج فضاء القشرة. ان تأثيرات استقطاب القلب لكثافة الانتقال حسبت بالأعتماد على شكل أنموذج Tassie الى جانب الصيغة الرياضية المشتقة للحالة الأرضية لتوزيعات كثافة الشحنة النووية. في هذا البحث ركزنا على الأنتقالات الكهربية $0_1^+3 \rightarrow 2_1^+3$ للنواة ^{50}Ti و $0_1^+1 \rightarrow 2_1^+1$ للنواة ^{50}Cr و $0_1^+2 \rightarrow 2_1^+2$ للنواة ^{52}Cr . أظهرت هذه الدراسة ان تأثيرات استقطاب القلب (والتي تمثل نمط تجميعي) تكون اساسية للحصول على توافق جيد بين الحسابات النظرية والنتائج العملية لعوامل التشكل الطولية $C2$ للأستطارة الألكترونية غير المرنة.

Introduction

Charge density distributions, transition densities and form factors are considered as fundamental characteristics of the nucleus. These quantities are usually determined experimentally from the scattering of high energy electrons by the nucleus. The information extracted from such experiments is more accurate with higher momentum transfer to the nucleus. Various theoretical methods [1- 3] are used for calculations of the charge density distributions (CDD), among them the Hartree-Fock method with the Skyrme effective interaction the theory of finite Fermi systems and the single particle potential method. Calculations of form factors [4] using the model space wave function alone is inadequate for reproducing the data of electron scattering. Therefore effects out of the model space, which is called core polarization effects, are necessary to be included in the calculations. These effects can be considered as a polarization of core protons by the valence protons and neutrons. Core polarization effects can be treated either by connecting the ground state to the J -multipole $n\hbar\omega$ giant resonances [4], where the shape of the transition densities for these excitations is given by Tassie model [5] or by using a microscopic theory [6-10] which permits one particle-one hole (1p-1h) excitations of the core and also of the model space to describe these longitudinal excitations. Comparisons between theoretical and observed longitudinal electron scattering form factors have long been

used as stringent test of models of nuclear structure.

In this study, derive an analytical form for the ground state charge density distributions of low mass fp shell nuclei ($40 \leq A \leq 56$) using the single particle wave functions of the harmonic oscillator and the occupation numbers of the states, which is determined from the comparison between theory and experiment.

This study is aimed to investigate the inelastic longitudinal electron scattering form factors, where the deformation in nuclear collective modes (which represent the core polarization effects) is taken into consideration besides the shell model space transition density. Core polarization transition density is evaluated by adopting the shape of Tassie model together with the derived form of the ground state charge density distribution. Our investigation is devoted on $0_1^+3 \rightarrow 2_1^+3$ transition in ^{50}Ti , $0_1^+1 \rightarrow 2_1^+1$ transition in ^{50}Cr and $0_1^+2 \rightarrow 2_1^+2$ transition in ^{52}Cr nuclei. It is found that the core polarization effects are essential for reproducing a remarkable agreement between the calculated inelastic longitudinal $C2$ form factors and those of experimental data.

Theory

The interaction of the electron with charge distribution of the nucleus gives rise to the longitudinal or Coulomb scattering. The longitudinal form factor is related to the CDD through the matrix elements of Coulomb multipole operators $\hat{T}_J^L(q)$ and is given by [4]

$$|F_J^L(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| \left\langle f \left\| \hat{T}_J^L(q) \right\| i \right\rangle \right|^2 |F_{cm}(q)|^2 |F_{fs}(q)|^2 \quad (1)$$

where J and Z are the multipolarity and the atomic number of the nucleus, respectively. $F_{cm}(q)$ is the center of mass correction, which remove the spurious state arising from the motion of the center of mass when shell model wave function is used and is given by [4]

$$F_{cm}(q) = e^{qb^2/4A} \quad (2)$$

where A is the nuclear mass number and b is the harmonic oscillator size parameter. $F_{fs}(q)$ is the free nucleon form factor and assumed to be the same for protons and neutrons and it takes the form [4].

$$F_{fs}(q) = e^{-0.43q^2/4} \quad (3)$$

The longitudinal operator is defined by [11]

$$\hat{T}_{J,t_z}^L(q) = \int dr j_J(qr) Y_J(\Omega) \rho(r, t_z) \quad (4)$$

where $j_J(qr)$ is the spherical Bessel function, $Y_J(\Omega)$ is the spherical harmonic wave function and $\rho(r, t_z)$ is the charge density operator. The reduced matrix elements in spin and isospin space of the longitudinal operator between the final and initial many particles states of the system including the configuration mixing are given in terms of the one body density matrix (OBDM) elements times the single particle matrix elements of the longitudinal operator [4]

$$\langle f \parallel \hat{T}_{JT}^L \parallel i \rangle = \sum_{a,b} \text{OBDM}^{JT}(i, f, J, a, b) \langle b \parallel \hat{T}_{JT}^L \parallel a \rangle \quad (5)$$

where the OBDM are calculated in terms of the isospin - reduced matrix elements.

The model space matrix elements is not adequate to describe the absolute

strength of the observed gamma-ray transition probabilities, because of the polarization in nature of the core protons by the model space protons and neutrons. The many particle reduced matrix elements of the longitudinal operator, consists of two parts one is for the model space and the other is for core polarization matrix element [6], i.e.

$$\langle f \parallel \hat{T}_J^L(\tau_z, q) \parallel i \rangle = \langle f \parallel \hat{T}_J^{msL}(\tau_z, q) \parallel i \rangle + \langle f \parallel \hat{T}_J^{corL}(\tau_z, q) \parallel i \rangle \quad (6)$$

where the model space matrix element has the form [12]

$$\langle f \parallel \hat{T}_J^{msL}(\tau_z, q) \parallel i \rangle = e_i \int_0^\infty dr r^2 j_J(qr) \rho_{J,\tau_z}^{ms}(i, f, r) \quad (7)$$

where $\rho_{J,\tau_z}^{ms}(i, f, r)$ is the transition charge density of model space and is given by [4]

$$\rho_{J,\tau_z}^{ms}(i, f, r) = \sum_{j,j' \in (ms)}^{ms} \text{OBDM}(i, f, J, j, j', \tau_z) \times \langle j \parallel Y_J \parallel j' \rangle R_{n_l}(r) R_{n_{l'}}(r) \quad (8)$$

$$\langle f \parallel \hat{T}_J^{corL}(\tau_z, q) \parallel i \rangle = e_i \int_0^\infty dr r^2 j_J(qr) \rho_{J,\tau_z}^{cor}(i, f, r) \quad (9)$$

where ρ_{J,τ_z}^{cor} is the core- polarization transition density which depends on the model used for core polarization. To take the core- polarization effects into consideration, the model space transition density is added to the core-polarization transition density that describes the collective modes of nuclei. The total transition density

becomes

$$\rho_{J\tau_z}(\mathbf{i}, \mathbf{f}, \mathbf{r}) = \rho_{J\tau_z}^{\text{ms}}(\mathbf{i}, \mathbf{f}, \mathbf{r}) + \rho_{J\tau_z}^{\text{cor}}(\mathbf{i}, \mathbf{f}, \mathbf{r}) \quad (10)$$

where $\rho_{J\tau_z}^{\text{cor}}$ is assumed to have the form of Tassie shape and given is by [5]

$$\rho_{J\tau_z}^{\text{cor}}(\mathbf{i}, \mathbf{f}, \mathbf{r}) = N \frac{1}{2} (1 + \tau_z) r^{J-1} \frac{d\rho(\mathbf{i}, \mathbf{f}, \mathbf{r})}{dr} \quad (11)$$

where N the proportionality constant.

In Eq. (11), $\rho(i, f, r)$ is the ground

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2} b^3} \times \left[5 - \frac{3}{2} \alpha + \frac{11}{3} \alpha \left(\frac{r}{b}\right)^2 + (4-2\alpha) \left(\frac{r}{b}\right)^4 + \left\{ \frac{8}{105} (z-20) + \frac{8}{3} \alpha \right\} \left(\frac{r}{b}\right)^6 \right] \quad (12)$$

The parameter α is determined from the central distribution $\rho(r=0)$ of Eq. (12)

$$\rho(0) = \frac{1}{\pi^{3/2} b^3} \left[5 - \frac{3}{2} \alpha \right] \quad (13)$$

where the value of $\rho(0)$ can be taken from the experiments. The mean square radii (*MSR*) of considered nuclei are obtained by

$$\langle r^2 \rangle = \frac{4\pi}{Z} \int_0^\infty \rho(r) r^4 dr \quad (14)$$

Using Eq. (13) in (14), we obtain:

$$\langle r^2 \rangle = \frac{b^2}{2} \left[7 - \frac{20}{Z} \right] \quad (15)$$

where the harmonic oscillator size parameter b is obtained by introducing the experimental *MSR* of considered nuclei into Eq. (15).

Results, discussion and conclusions

The *C2* longitudinal form factors of the low mass *fp* shell nuclei are

state *CDD* for the low mass *fp* shell nuclei. It is derived on the assumption that there is a core of filled *1s, 1p, 1d*, shells and the occupation numbers of protons in *2s, 1f* and *2p* shells are equal to $(2-\alpha)$, $(z-20-\alpha)$ and α , respectively. The parameter α characterizes the deviation of the occupation numbers from the prediction of the simple shell model ($\alpha = 0$). Using this assumption together with the single particle wave functions of the harmonic oscillator potential, the ground state charge density distribution for the low mass *fp* shell nuclei is obtained as

calculated for the transition $J_i^\pi T_i = 0_1^+ T$ to $J_f^\pi T_f = 2_1^+ T$ in ^{50}Ti and $^{50,52}\text{Cr}$ nuclei. The deformation in nuclear collective modes (which represent the core polarization effects) is taken into consideration by using the shape of Tassie model [5], that depends on the ground state *CDD*. The ground state *CDD* for the low mass *fp* shell nuclei ($40 \leq A \leq 56$), which is used in Tassie model, is derived and formulated in terms of the single particle wave functions and the occupation numbers of the states as is given by Eq. (12). These occupation numbers are determined from the comparison between the calculated and experimental *CDD*. The single particle wave functions are those of the

harmonic oscillator potential whose oscillator size parameters b are chosen in such a way that to reproduce the experimental root mean square (*rms*) charge radii $\langle r^2 \rangle_{\text{exp}}^{1/2}$. The experimental *CDD* at $r=0$, $\rho_{\text{exp}}(0)$

[13], are used in Eq. (15) to determine the values of the parameter α for all considered nuclei. The experimental values of $\langle r^2 \rangle_{\text{exp}}^{1/2}$, and $\rho_{\text{exp}}(0)$, for considered nuclei are given in Table 1 whereas the calculated values of b , α and the proton occupation numbers of $2s$ and $2p$ shells for ^{50}Ti and $^{52,50}\text{Cr}$ nuclei are given in Table 2.

Inelastic longitudinal $C2$ form factors for the transitions $J_i^\pi T_i = 0_1^+ T$ to $J_f^\pi T_f = 2_1^+ T$ of considered nuclei are displayed in Fig. 1. The dashed curves represent the contribution of the model space where the configuration mixing is taken into account and the solid curves represent the total contribution, which is obtained by taking the model space together with the core polarization effects. The experimental data of Ref. [14] are represented by solid circles. It is very clear from Fig. 1 that the core-

polarization effects give a strong modification to the form factors, where the core polarization effects enhance the $C2$ form factors at the first and second maximum and bring the calculated values very close to the experimental data. Besides, the locations of the diffraction minimum, in all considered nuclei, are located in the correct places when the core polarization effects are included in the calculations. However, the available experimental data [14] presented in Fig. 1 is very well reproduced by the present calculations throughout all momentum transfer values. It is concluded that the core polarization effects, which represent the collective modes, are essential in obtaining a remarkable agreement between the calculated $C2$ longitudinal form factors of the low mass fp shell nuclei and those of experimental data.

Table 1: The experimental values of $\langle r^2 \rangle_{\text{exp}}^{1/2}$ (in unit of fm), and $\rho_{\text{exp}}(0)$ (in unit of $e \cdot fm^{-3}$) used in the present calculations.

Nuclei	$\langle r^2 \rangle_{\text{exp}}^{1/2}$ [13]	$\langle r^2 \rangle_{\text{theo}}^{1/2}$	$\rho_{\text{exp}}(0)$ [13]
^{50}Ti	3.571	3.492	0.0750351
^{50}Cr	3.612	3.521	0.0775860
^{52}Cr	3.613	3.5	0.0765349

Table 2: The calculated values of b (in unit of fm), α and the proton occupation numbers of $2s$ and $2p$ shells.

Nuclei	b	α	Occupation No. of $2s$ shell	Occupation No. of $2p$ shell
^{50}Ti	2.053	0.932160	1.0702865	0.9297135
^{50}Cr	2.040	0.881256	1.1187465	0.8812535
^{52}Cr	2.050	0.884172	1.1158276	0.8841724

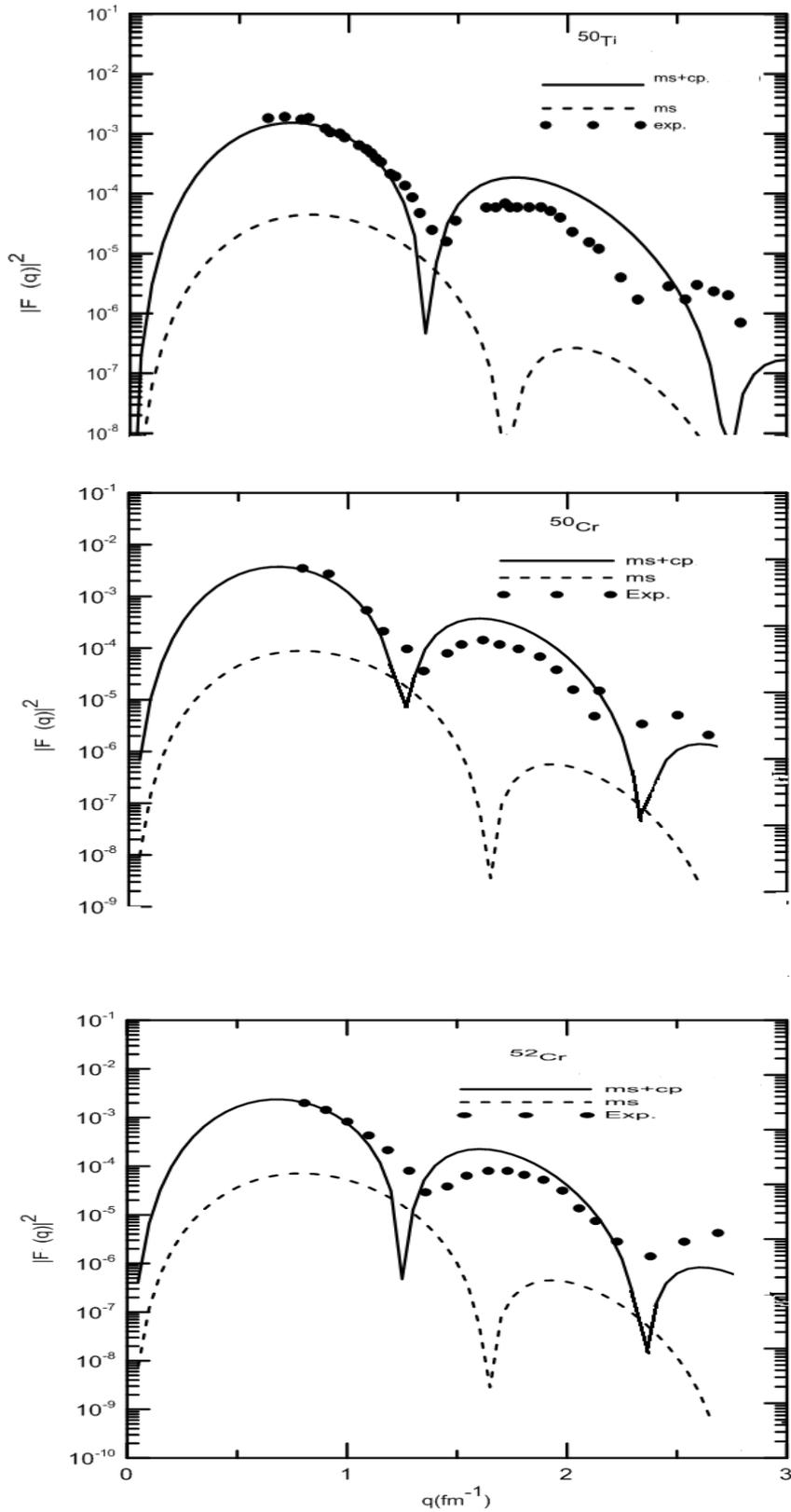


Fig. 1: Inelastic longitudinal C2 form factors for the transitions $J_i^\pi T_i = 0_1^+ T$ to $J_f^\pi T_f = 2_1^+ T$ in ^{50}Ti and $^{52,50}\text{Cr}$ nuclei. The dashed curves represent the contribution of the model space and the solid curves represent the total form factors obtained by the sum of model space and core polarization contributions. The experimental data of Ref. [14] are represented by solid circles.

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