Evaluated the level density for proton induced nuclear resonances in (P+^{48}Ti) reaction using different models

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Abstract

The experimental proton resonance data for the reaction P+^{48}Ti have been used to calculate and evaluate the level density by employed the Gaussian Orthogonal Ensemble, GOE version of RMT, Constant Temperature, CT and Back Shifted Fermi Gas, BSFG models at certain spin-parity and at different proton energies. The results of GOE model are found in agreement with other, while the level density calculated using the BSFG Model showed less values with spin dependence more than parity, due the limitation in the parameters (level density parameter, a, Energy shift parameter, E_1 and spin cut off parameter, σ_c). Also, in the CT Model the level density results depend mainly on two parameters (T and ground state back shift energy, E_0), which are approximately constant in their behavior with the proton energy compared with GOE model. The RMT estimation used to calculate the corrections of the incompleteness proton resonance measurement data by using two methods; the conventional analysis method, which depends on the resonance widths and the updated, developed, tested and applied a new analysis method, which depends mainly on the resonance spacings. The spacing analysis method is found much less sensitive to non-statistical phenomena than is the width analysis method. Where the analysis of a given data set via these two independent analysis methods indicated the increasing in the reliability of the determination of the missing fraction of levels, the observed fraction f between $0.87^{+0.13}_{-0.11} - 0.68^{+0.12}_{-0.12}$ for different spin-parity of this reaction and then the distinguishability in the level density calculations can be achieved. The modified Porter Thomas distribution along with the maximum likelihood function have been used to get the missing levels corrections for 5 proton resonance sequences in the present reaction. To estimate the present long-range correlations for pure sequence of levels the mean square of the deviation of the cumulative number of levels from a fitted straight line represented by the Dyson and Mehta Δ_3 statistic has been employed for spin parity $\frac{1}{2}^+$, and calculated the $<\Delta_3>$ against the cumulative number of proton levels.

تقييم كثافة المستوي لبروتون يحرض رنين نووي للتفاعل (P+^{48}Ti) باستخدام نماذج متعددة

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الخلاصة

استخدمت بيانات رنين البروتون العملية للتفاعل P+^{48}Ti لحساب وتقييم كثافة المستوي باستخدام نماذج GOE مجاميع جاوس المتعددة، BSFG الازاحة الخلفية لغاز فرمي و CT النموذج الثابت لدرجة الحرارة، عند برم ـ تماثل معين ولطاقة بروتون متغيرة. وجد ان نتائج نموذج GOE تتوافق مع الاخرين لنفس النموذج ، بينما بينت كثافة المستوي المحسوبة لنموذج BSFG ذات قيم اقل اعتمادا على البرم اكثر من التماثل وذلك بسبب المحددات في المعلمات (a, E_1 and σ_c). كذلك، عند النموذج CT أعتمد نتائج كثافة المستوي اساسا على المعلمات (T and E_0)، والتي وجدت ان قيمها ثابتة التصرف مع طاقة البروتون مقارنة مع نموذج GOE. في تقدير RMT استخدم حساب تصحيحات البيانات المقاسة لرنين البروتون غير المتكاملة باستخدام طريقتين: الاولى طريقة التحليل الاعتيادية والتي تعتمد على عرض الرنين والثانية التحليل المطور والمحور والذي وجد ان طريقة التحليل الفضائي اقل حساسية الى الظاهرة غير الاحصائية منها الى طريقة تحليل العروض. بينما بين التحليل لمجموعة البيانات باستخدام هذه الطريقتين امكانية ايجاد الجزء المفقود من المستويات، حيث لوحظ ان الجزء المرئي ضمن حدود $0.87^{+0.13}_{-0.11} - 0.68^{+0.12}_{-0.12}$ الى برم-تماثل مختلف لهذا التفاعل ومنها يمكن تمييز امكانية حساب كثافة المستوي. تم استخدام توزيع بورتر-ثوماس المعدل مع دالة الاحتمالية العظمى لإيجاد تصحيحات مستويات مفقودة ولسلسلة متكونة من خمس رنين بروتون في التفاعل الحالي. لغرض تقدير التلازم الطويل لسلسلة نقية من المستويات الرنينية تم الاستعانة بإحصاء مهاتا Δ_3 لنموذج برم-تماثل $\frac{1}{2}^{+}$ مع العدد الكلي لمستويات البروتونات الجديدة.

Introduction

Nuclear level densities (NLDs) are of special importance in predicting the distribution of all excited levels of a nucleus that presents a great challenge to our understanding of this complicated quantum system, compound nucleus. On the other hand, NLDs represent a very important ingredient in statistical model calculations of nuclear reaction cross section, which are needed in many applications from astrophysical calculations (determining thermos-nuclear rates for nucleosynthesis) to fission or fusion reactor designs [1].

The theoretical research of level density started with the pioneering work of Bethe [2]. He has obtained a simple level density formula for a gas of non-interacting fermions with equally spaced non degenerate single particles. This formula based on the Fermi gas or the so- called Back Shifted Fermi Gas model (BSFG) and simple counting arguments.

The independent particle model, in which nucleons are assumed to move freely in an average potential, has been successful predicted the bound state structure of various nuclei [3, 4]. In this model the odd-even effects are included by means of a pairing energy shift, and one can solve the equation and determine the nuclear levels.

However, as the excitation energy is increased, the number of possible configurations becomes very large, and the exact solution of the Schrodinger equation is become difficult. It is under these circumstances the quantum structure also is very complicated (i.e. high level density) [5]. The formula proposed by Gilbert and Cameron [6] combined the BSFG formula at high excitation energies with the Constant Temperature (CT) formula for lower energies. The back-shift parameter E_1 in the BSFG model was replaced by the pairing force V_0. Through fitting procedure, the parameters (level density parameter (a), back-shift

parameter (E_1), pairing force (V_0), and spin cutoff parameter (σ_C), in both regions, are erected to match with the experimental data. But from the evaluation of these models with the experimental work, some of the levels are missing, in any experimental data set, which cannot explicate the data clearly, especially in nucleon resonance regions. Therefore, a correction method is needed before determining the level densities from these regions. The standard method for the missing level correction involves resonance widths, where, from this approach one can assume the resonance widths obey a Porter-Thomas distribution (PTD) and that all levels below some cut-off widths are not observed. This approach works fairly well but it has several limitations. In order to determine the most reliable correction for missing levels, an additional method is needed. Since the PTD follows from Random Matrix Theory (RMT) [7], the nearest neighbor spacings idea, for a set of states with the same symmetry properties that obey the Wigner distribution, can be predicted easily by RMT. In this theory the widths and spacings are not correlated and then the analysis of the spacing distribution can provide an independent determination of the missing fraction of levels.

On the basis of fits to all known bound levels up to a certain energy and to the resonance region level densities of 87 nuclei, ref [8], and for 310 nuclei, ref [1] have been implemented both the BSFG and the CT models work equally well in reproducing experimental densities. Also, ref [9] used nuclear level schemes and neutron resonance densities for 75 nuclides to determine parameter of level density formula, the spacing distribution of levels with equal spin and parities, and spacing correlations.

They found that the spacing distribution lied between the Poisson and Wigner distributions.

In RMT ref [5] applied the width and spacing analysis methods to 15 protons and 2 neutrons resonance data sequences and they observed fraction of levels for each sequence. Values for level density and strength function for each sequence are obtained and the spin and parity dependence of the level densities is considered. The parity and completeness of ^{45}Sc data were tested via several statistical analyses by [10]: the nearest neighbor spacing distribution, the reduced width distribution and the Dyson-Mehta $\Delta 3$ statistic. The basic concepts of RMT is discussed by [11] using the ensemble of random matrices originally introduced by Wigner, the Gaussian ensemble as a starting point. Also, the correlation between NLD parameters of the BSFG and CT models have been investigated by [12, 13] for different nuclei and the calculated NLDs have been compared with experimental works.

Theoretical models, NLD

Understanding the NLD, through the calculation of neutron and proton capture reaction rates, expanded the sight of applications in, say, the accelerator-driven transmutation of nuclear waste and radio chemical analysis [9]. For excitation energies not much higher than the neutron separation energy, one can use simple models with only two parameters for the level density, such as the BSFG or the CT models [3]. Where, the NLD, dependence on the excitation energy E and spin J, is assumed to have a separable form [1]:

$$\rho(E, J, \pi) = \frac{1}{2}\rho(E)f(J) \qquad (1)$$

where f (J) is spin distribution function in terms of spin cutoff parameter σ_c, and it is given by:

$$f(J) = e^{-J^2/2\sigma c^2} - e^{[-(J+1)^2/2\sigma c^2]} \quad (2)$$

The BSFG and CT models

The simplest system which does not have equidistant levels is the Fermi gas (FG) system, where the single particle level density increases with the square root of the kinetic energy of the particles. This model historically had the largest impact on the interpretation of experiments that was used by Bethe [1]. The BSFG formula was proposed by many authors and also they adopted the FG formula with the shifted ground state position (Δ) and the level density parameter (a), as parameters to be adjusted to the experimental results [14,15] . In this model the level density depends on two parameters a and E_1:

$$\rho_{BSFG}(\text{E}) = \frac{e^{2\sqrt{a(E-E1)}}}{\sigma c\; 12\sqrt{2}a^{1\backslash4}(E-E1)^{5\backslash4}} \qquad (3)$$

Where, ($E-E_1 = at^2$) is the effective excitation energy [5], t is the thermodynamic temperature [14] and the spin cut off parameter:

$$\sigma c^2 = 0.0888\; A^{\frac{2}{3}}\sqrt{a(E-E1)} \qquad (4)$$

where A represent the mass number. The bound states alone and the bound states plus resonance density were also fit with the CT model formula relating the nuclear temperature (T), which is different from the thermodynamic temperature (t), to the level density[16]:

$$\rho_{CT}(\text{E}) = (\frac{1}{T})e^{(E-E_0)/T} \qquad (5)$$

where E_0 is the back shift energy [1], which is based on the C_0 value, given by [9]:

$$C_0 = -(12\pm3)A^{-(0.32\pm0.05)} \qquad (6)$$

where, $C_0 = E_0 - \Delta$

In the thermodynamically approach, the nuclear temperature is defined by the relation [17]:

$$\frac{1}{T} = \frac{1}{\rho}\frac{d\rho}{dE} = \frac{1}{T} = \frac{1}{\sqrt{E/a}} - \frac{5}{4E} \qquad (7)$$

Gaussian orthogonal ensemble (GOE) model:

Width analysis of imperfect sequences

The Gaussian assumption for the distribution of reduced width amplitudes leads to the PT distribution for the dimensionless strength parameter y:

$$p(y) = \frac{1}{\sqrt{2\pi y}}e^{\frac{-y}{2}} \qquad (8)$$

where, $y = \gamma^2/<\gamma^2>$, γ^2 Is the reduced width and $<\gamma^2>$ is the average reduced width.

One assumes that all levels weaker than the weakest observed level are missed and that all levels with larger widths are observed. A modified PT distribution was introduced by Fröhner [18]. According to this distribution, the smallest widths are most frequent. Because these weak levels may not be observed, an experimental level sequence is usually incomplete, and therefore the width distribution is distorted. While the absence of weak levels causes the sequence to be incomplete. There are various other effects that also can distort the observed width distribution. Non-statistical phenomena such as doorway states can affect the sequence. Another cause is spin misassignments. This leads to an impure sequence, which will have a different distribution. One must consider these effects when analyzing the observed resonance widths [5]. As shown in Figs. 1-5 the experimental observed reduced widths and number of proton resonance for the reaction P+^{48}Ti at different proton energy range 3-4 MeV, taken from ref[5], for different spin-parity.
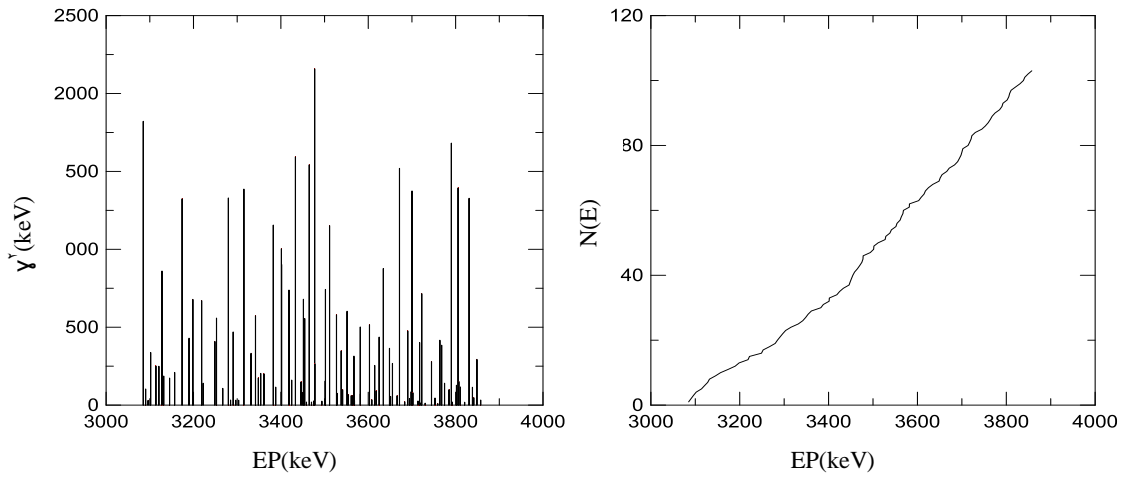
Fig.1: The first figure shows the reduced width, the second figure shows number plot for 103 observed $\frac{1}{2}^{+}$ ***resonances.***
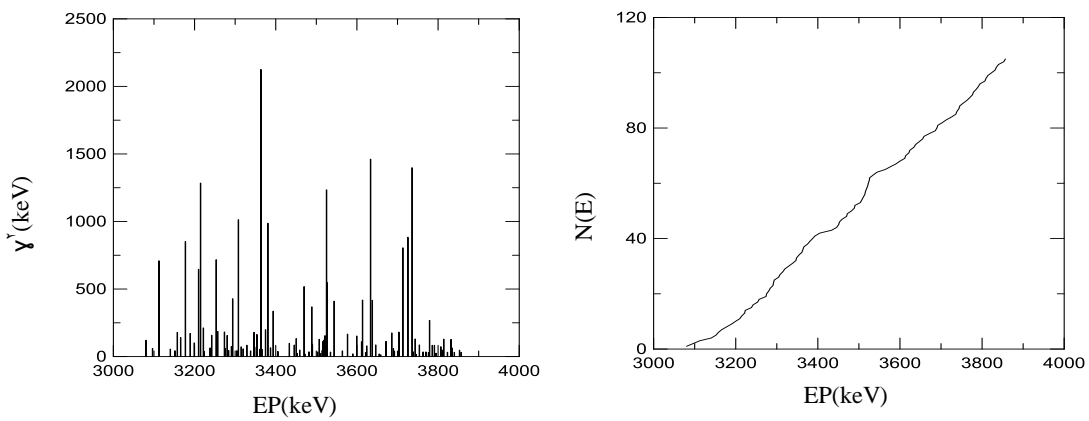

Fig.2: The first figure shows the reduced width, the second figure shows number plot for 105 observed $\frac{1}{2}^{-}$ ***resonances at different proton energy.***
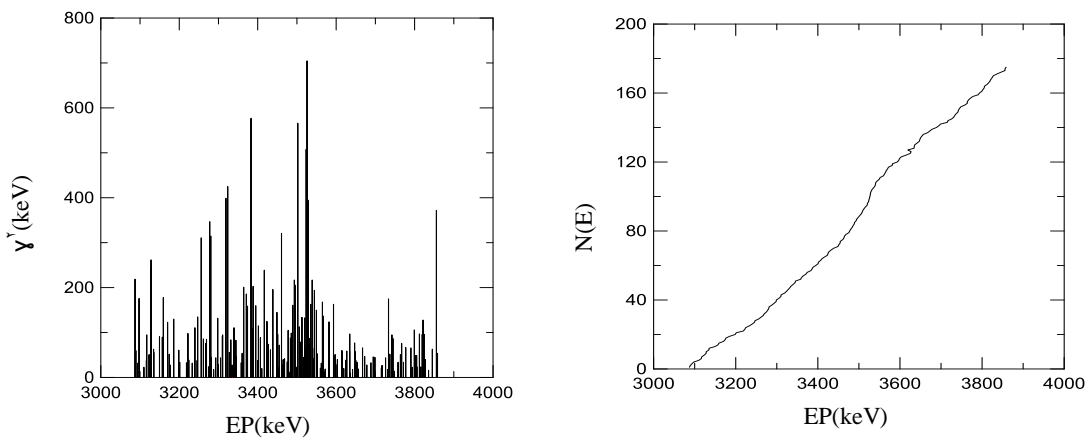

Fig.3: The first figure shows the reduced width, the second figure shows number plot for 175 observed $\frac{3}{2}^{-}$ ***resonances at different proton energy.***
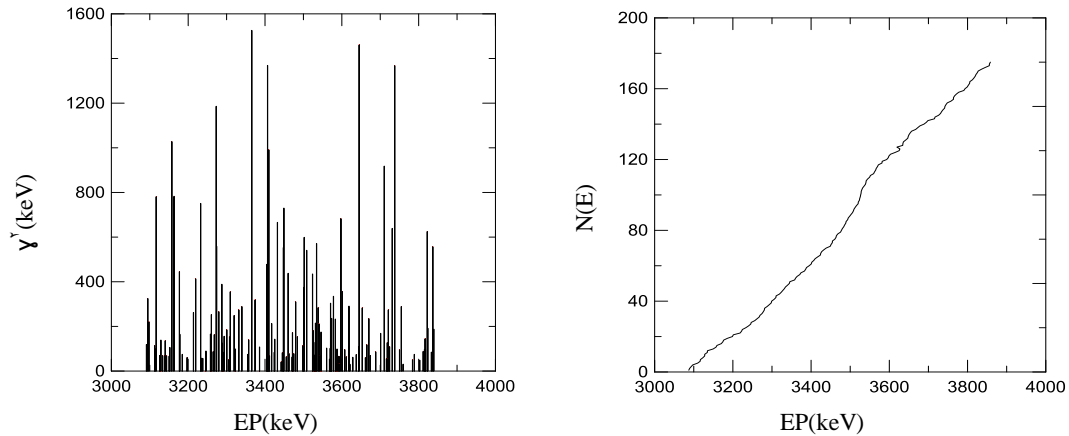
Fig.4: The first figure shows the reduced width, the second figure shows number plot for 139 observed $\frac{3}{2}^+$ resonances at different proton energy.
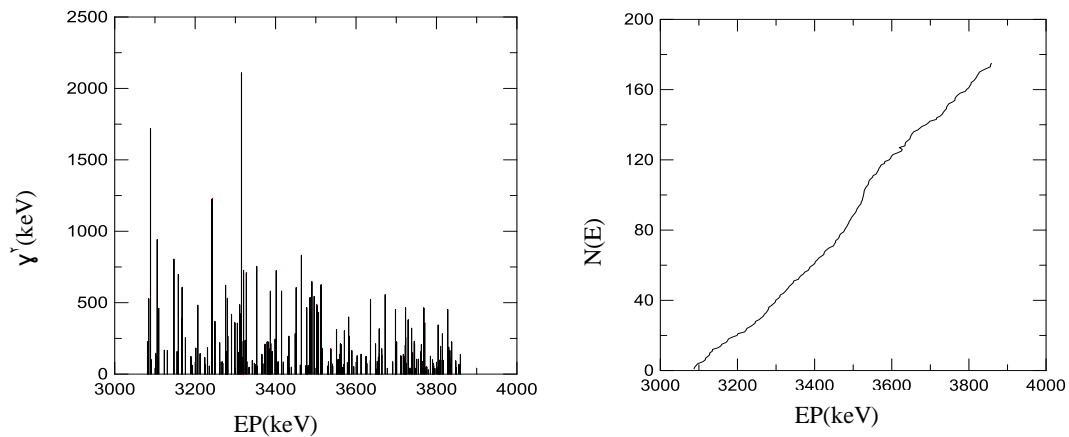

Fig.5: The first figure shows the reduced width, the second figure shows number plot for 180 observed $\frac{5}{2}^+$ resonances at different proton energy.

The standard iterative procedure

Most of the levels that are missed are below the threshold of experimental observability in a particular experiment. Therefore, the simplest assumption is normally adopted.

One assumes that all of the levels with reduced widths smaller than the minimum observed reduced width are not detected and that all resonances with widths larger than the minimum value are observed. Usually the cut-off parameter y_0 is taken to be the smallest of all the observed widths divided by the average reduced width. The observed average reduced width of a sequence of given J^π is

$$<\gamma^2>_{obs} = \sum_{i=1}^{Nobs} \frac{\gamma_1^2}{N_{obs}} \qquad (9)$$

And the cut-off for that sequence is

$$y_0 = \frac{\gamma^2_{min}}{\langle\gamma^2\rangle_{obs}} \qquad (10)$$

The observed fraction f of the sequence is obtained by

$$1\text{-}f = \int_0^{y_0} P(y)dy \qquad (11)$$

Where P(y) is the PT distribution and $(1 - f)$ is the fraction of levels missed. The number of observed levels N_{obs} must be corrected by this missing fraction. The corrected number N_{new} is closer to the true number of levels.

Because of missing levels, the observed strength is smaller than the actual strength. The observed strength f_s can be found from

$$1 - f_s = \int_0^{y_0} y \, P(y) dy \qquad (12)$$

The observed strength should be corrected by 1-f_s giving a new value for the strength. The next step is to calculate the average reduced width with the new strength and the new number of levels. Then one can recalculate the cut-off for the sequence using a new average reduced width, recalculate 1 - f and 1 - f_s with the new cut-off, and then repeat the entire process. This procedure is repeated until the missing fraction approaches a constant. Because the missing fraction is obtained with several iterations, this method is called the iterative method. A new corrected number of levels in the sequence are determined from

$$N_{new} = \frac{N_{obs.}}{f} \qquad (13)$$

and the total strength is corrected

$$\sum \gamma_i^2 = \frac{\sum_{obs} \gamma_i^2}{f_s} \qquad (14)$$

These values are used to determine a new average reduced width

$$\langle \gamma^2 \rangle_{new} = \frac{\sum \gamma_i^2}{N_{new}} \qquad (15)$$

Using this new average reduced width, a new value of y_0 is defined, and the above steps are repeated. After a few iterations a constant value of 1 - f is obtained.

The corrected number of levels is

$$N = \frac{N_{obs}}{f} \qquad (16)$$

One can determine the average level spacing D or level density ρ from [5, 20]:

$$D = \frac{1}{\rho} = \frac{E_{max} - E_{min}}{N-1} \qquad (17)$$

Spacing analysis method

The nearest-neighbor spacing's distribution of a perfect GOE sequence (NNSD) are described by the Wigner distribution

$$P_{GOE}(x) = \frac{\pi}{2} x e^{-\pi x^2/4} \qquad (18)$$

Here $x \equiv S/D$, where S is spacing between adjacent levels and D is the average spacing.

The experimental value of the average spacing D_{obs} differs from the true value D according to D= f D_{obs}, where *f* is the fraction of levels observed experimentally. We define a variable Z≡ S_{obs}/D_{obs}, where S_{obs} is spacing between adjacent observed levels, and D_{obs} is the average *observed* spacing.

The variables *x* and *z* are related by $z = f \, x$.

The spacing distribution for NNSD can be written as:

$$P(z) = \sum_{k=0}^{\infty} a_k \, \lambda P(k; \lambda_z) \qquad (19)$$

The parameters a_k give the relative contributions of the *k*-th NNSD *P* $(k; \lambda_z)$, λ is a parameter that characterizes the incompleteness of the sequence. Therefore, from the constraints of the parameter $a_k = f(1-f)^k$ and $\lambda = 1/f$ and the equation (19) become:

$$P(x) = \sum_{k=0}^{\infty} f(1-f)^k p(k; x) \qquad (20)$$

For $f = 1$ this reduces to the Wigner distribution $P(0; x)$, Eq. (18), [21].

The integrated level density N(E) or cumulative number of proton levels can be written as [5]:

$$N(E) = \int_0^E \rho(E) dE = \exp(E - E_0)/T - \exp(-E_0/T) + N_0 \qquad (21)$$

The maximum likelihood (MLH) method

Now we are going to seek the value of the parameter vector that maximizes the likelihood function (MLF) [22], through considering the uncertainty in the missing fraction a modified Porter-Thomas distribution,

$$P_f(y) = \begin{cases} 0 & : y < y_0 \\ \frac{1}{erfc(\sqrt{y_0/2})} \frac{e^{-y/2}}{\sqrt{2\pi y}} & : y \geq y_0 \end{cases} \qquad (22)$$

This distribution goes to zero at values of y≤ y_0, reflecting the assumption that weak levels are not observed, and becomes the Porter-Thomas

distribution when $y \geq y_0$. We will refer to this distribution as the truncated Porter-Thomas distribution. The truncated Porter-Thomas distribution is normalized to unity when integrated over $y = [y_0, \infty]$. With this distribution one can construct the MLF as:

$$L = \prod_i P(y_i) \qquad (23)$$

where $y = \frac{\gamma^2}{\langle \gamma^2 \rangle}$, the uncertainty in $\langle \gamma^2 \rangle$ is obtained by calculating the MLF or equivalently the maximum of the natural logarithm of the function (MNLF) which is located at the most likely value of $\langle \gamma^2 \rangle$. The MNLF is normalized to unity and the value of $\langle \gamma^2 \rangle$ where the normalized likelihood function $(lnL)_{norm}$ equals to one half of its maximum value is used for a determination of the uncertainty in $\langle \gamma^2 \rangle$, which in turn is used for the uncertainty in f [5].

The Δ_3 statistic

The Δ_3 statistic developed by Dyson and Mehta is a popular measure of long-range correlations. It is defined as the mean square of the deviation of the cumulative number of levels from a fitted straight line [23]:

$$\Delta_3(L) \equiv \frac{1}{L} < \min_{a,b} \int_{x_0}^{x_0+L} dE \, [N(E) - aE - b]^2 > \qquad (24)$$

where the average is over the starting points X_0.

The Δ_3 (L, x) have been computed in the present work for each starting point ranging over the entire spectrum, and average over them. This has been done with idea that different intervals are considered do not overlap as to ensure that each contribution to the average is statistically independent [11,24].

Results and discussion

To ensure the evaluation of the GOE, CT and BSFG models, the experimental data of ref.[5] for the reaction P+[48]Ti at certain spins, have been used and compared the calculated level density with these models.

As shown in table 1 the level densities for P+[48]Ti width analysis have been investigated accurately using the GOE, CT and BSFG models. Where the GOE Model compared with refs [5,19] results and they are closed to each other with small errors mentioned in Table 1.The data has been sketched in fig. 6A and B, where the level densities for positive and negative parity are indicated by colored points. While the level density calculated by BSFG Model, with two parameters (a and E_1) and another parameter (σ_c) dependence, shows less values with spin dependence more than parity. Also, in the CT Model the level density results, depends on two parameters (T and E_0) are approximately constant in their behavior with the proton energy, as shown in Fig.6 (A and B).

Table 1: Level densities for P+[48]Ti via width analysis (GOE), constant temperature and back shifted Fermi gas model.

J^π [5]	N_{obs} [5]	$E_P$range (MeV)	ρ_{GOE} $(MeV)^{-1}(PW)^*$	ρ_{GOE} $(MeV)^{-1}$[19]	ρ_{GOE} [5]	ρ_{CT} $(MeV)^{-1}$	ρ_{BSFG} $(MeV)^{-1}$
$1/2^+$	103	3.0850-3.8574	152±20	151±10	152±13	15.416±17	50.168±17.39
$1/2^-$	105	3.0802-3.8568	168±34	166±16	159±17	15.409±15	50.138±17.38
$3/2^-$	175	3.0873-3.8584	314.5±27	308±21	295±22	15.427±26	74.627±26.86
$3/2^+$	139	3.0913-3.8395	271±88	254±21	251±22	15.229±15	73.134±26.38
$5/2^+$	180	3.0816-3.8595	339±39	318±21	313±24	15.438±15	68.316±26

(**pw**)*: **Present work**

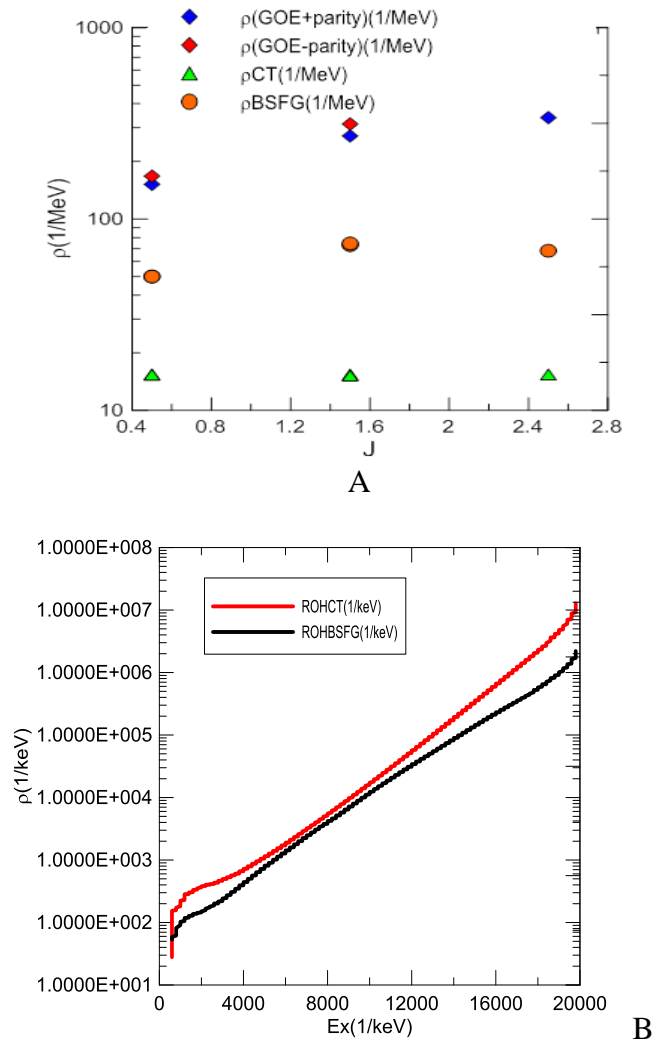***Fig.6: A- The J^π dependence of level density in P+⁴⁸Ti for GOE, BSFG and CT models. B- The level density as a function of the excitation energy for CT and BSFG models.***

The proton induced nuclear resonance has been studied according to the predictions of the GOE version of RMT. The RMT estimation used to calculate the corrections to the nucleon resonance data. As no measurement is without errors, data must be corrected for the incompleteness of the measurement. Two ways are used in the present work, the first one is to upgrade the conventional analysis method depends on the resonance widths. In the second one a development, testing and applying anew analysis method depends on the resonance spacing. When the two methods disagree, it is best to consider the data on a case-by-case basis. However, the spacing analysis method is much less sensitive to non-statistical phenomena than the width analysis method. Analysis of a given data set via these two independent analysis methods increased the reliability of the determination of the missing fraction of levels.

Conclusions

Since the widths follow the porter Thomas distribution, the weakest widths cannot be seen easily in experiments; therefore, this distribution can be rearranged to indicate the errors in data. Using the modified porter Thomas distribution along with the maximum likelihood function, one can get the missing levels corrections for 5 proton resonance sequences in the present reaction.

During the systematic calculated procedures and keeping in mind the existing of the possible effects of spurious levels, one might be tempted to accept the missing fraction idea, which determined from the NNSD. This gives an observed fraction f between $0.87^{+0.13}_{-0.11}$ − $0.68^{+0.12}_{-0.12}$, for different spin-parity of this reaction, as shown in Figs.7 and 8. This was agreeing with refs [5, 19]. The new cumulative number of proton levels as a function of proton energy of the p+^{48}Tireaction for different J^{π} shown in Fig.9, where the red line shows the

expected behavior if the average spacing over the entire energy range was equal to its value in the range Ep = 9.8-10.6 MeV. The mean square of the deviation of the cumulative number of levels from a fitted straight line represented by the Dyson and Mehta Δ_3 statistic, which is measured the long-range correlations for pure sequence of levels. Typical sketches shown in Fig. 10 for Δ_3 against L, for spin parity $\frac{1^{+}}{2}$, and the $<\Delta_3>$ against the cumulative number of proton levels.

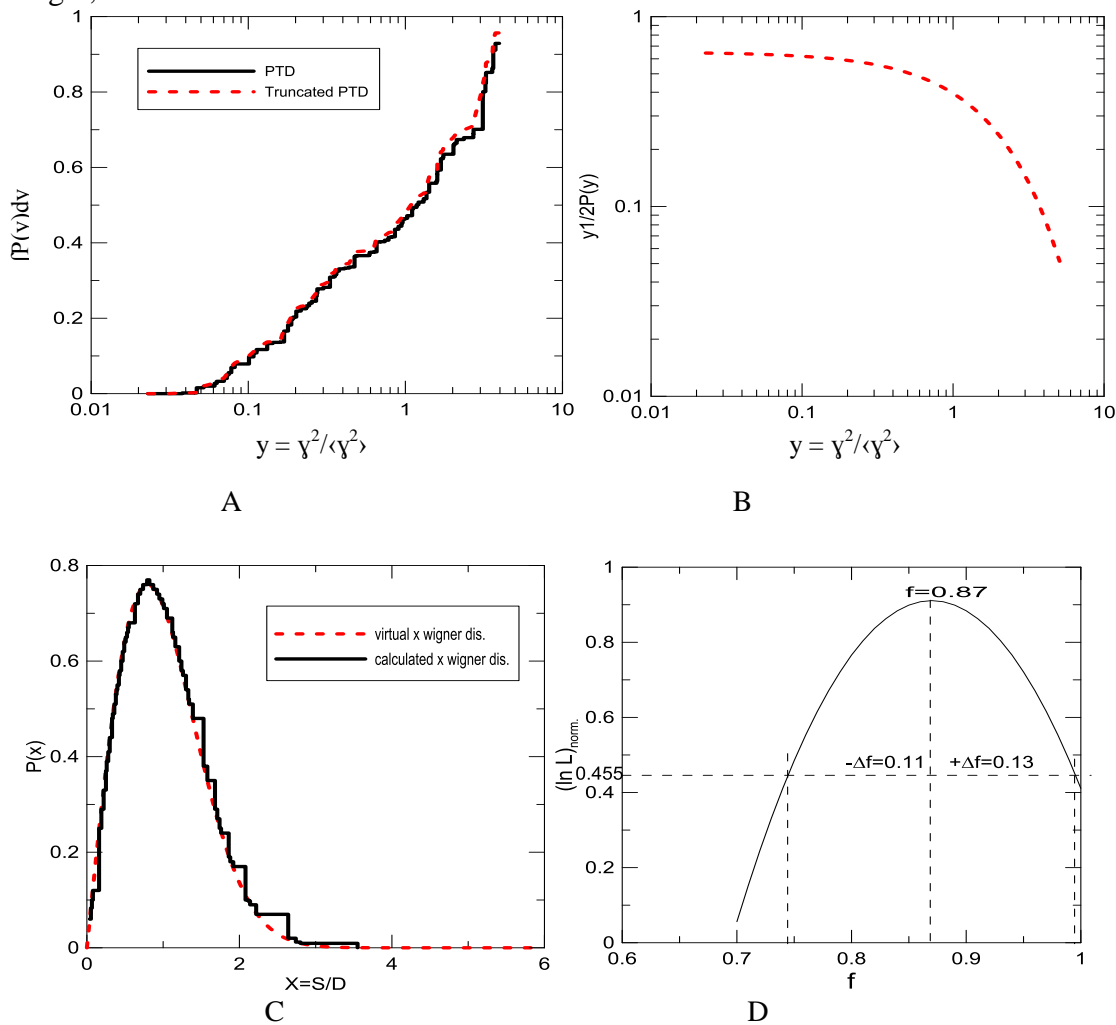
Fig.7: A- The cumulative probability of reduced widths for p +^{48}Ti ($\frac{1^{+}}{2}$) resonances as a function of the dimensionless parameter y and compared with the truncated Porter-Thomas result (red dashed curve). B- The truncated Porter-Thomas distribution. C- The probability distribution for virtual value of x (red dashed curve) and compared it with the calculated x (black solid curve). D- The MLF as a function of the observed fraction (f) of levels.
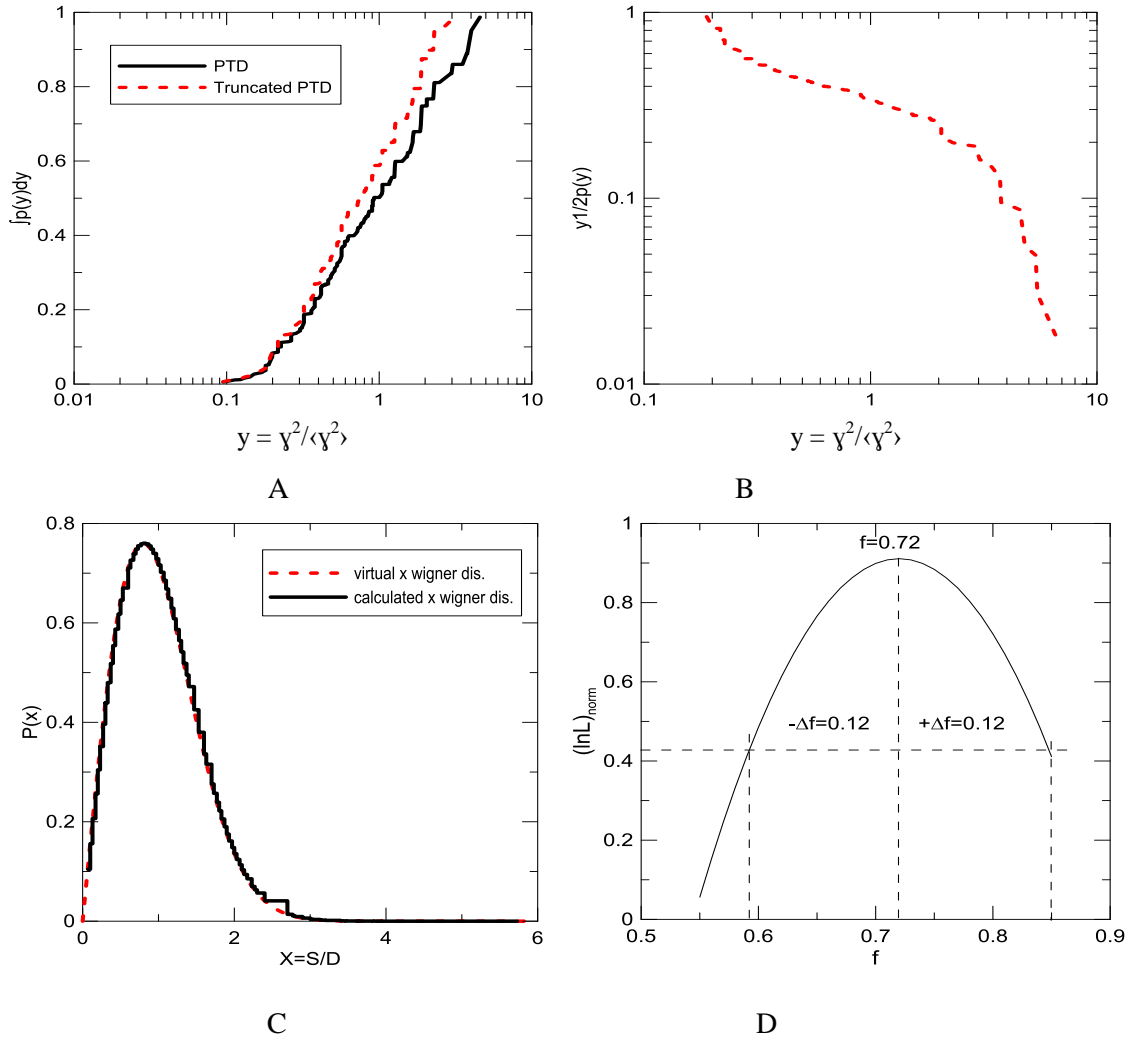
Fig.8: A- *The cumulative probability of reduced widths for p +^{48}Ti ($\frac{3}{2}^{-}$) resonances as a function of the dimensionless parameter y and compared with the truncated Porter-Thomas result (red dashed curve).* **B-** *The truncated Porter-Thomas distribution.* **C-** *The probability distribution, Wigner distribution, for virtual value of x (red dashed with the calculated x (black solid curve).* **D-** *The MLF as a function of the observed fraction (f) of levels.*
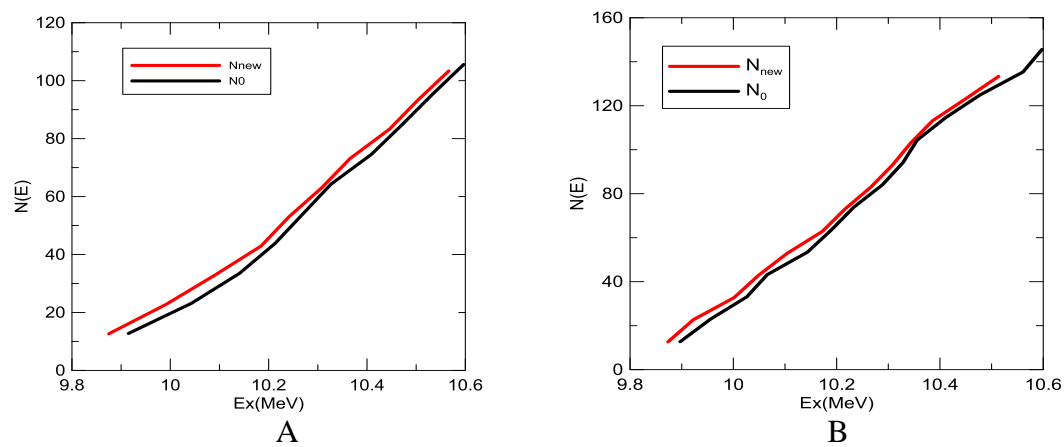

Fig.9: *The cumulative number of proton levels as a function of proton energy for p+^{48}Ti reaction compared with observed number of proton resonances and for different J$^{\pi}$, (A) $\frac{1}{2}^{+}$, and (B) $\frac{3}{2}^{-}$.*
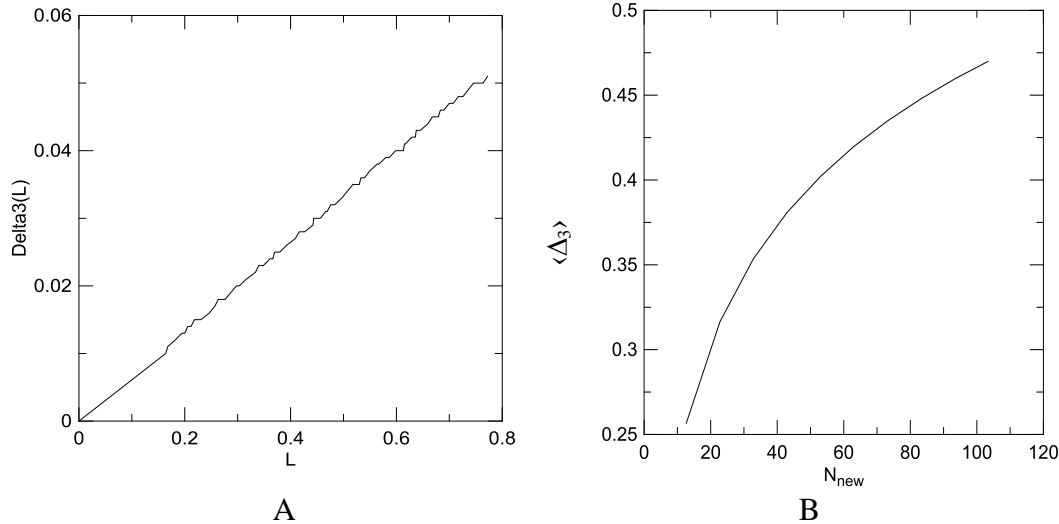
Fig.10: A - The Δ_3 statistics of Gaussian orthogonal ensemble as a function of L, B-The average value of Δ_3 as a function of new number of levels.

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