

Charge density distributions and electron scattering form factors of ^{19}F , ^{27}Al and ^{25}Mg nuclei

Raghad Imad Mahmood and Gaith Naima Flaiyh

Department of Physics, College of Science, Baghdad University, Baghdad, Iraq

E-mail: Rag_emed@yahoo.com

Abstract

An effective two-body density operator for point nucleon system folded with two-body correlation functions, which take account of the effect of the strong short range repulsion and the strong tensor force in the nucleon-nucleon forces, is produced and used to derive an explicit form for ground state two-body charge density distributions (2BCDD's) and elastic electron scattering form factors $F(q)$ for ^{19}F , ^{27}Al and ^{25}Mg nuclei. It is found that the inclusion of the two-body short range correlations (SRC) has the feature of reducing the central part of the 2BCDD's significantly and increasing the tail part of them slightly, i.e. it tends to increase the probability of transferring the protons from the central region of the nucleus towards its surface and to increase the root mean square charge radius $\langle r^2 \rangle^{1/2}$ of the nucleus and then makes the nucleus to be less rigid than the case when there is no (SRC). It is also found that the effects of two body tensor correlations (TCs) on 2BCDD's and $\langle r^2 \rangle^{1/2}$ are in opposite direction to those of (SRC).

Key words

Elastic electron scattering, Charge density, Root mean square charge radii.

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توزيعات كثافة الشحنة وعوامل التشكل للأستطارة الألكترونية للنوى ^{27}Al و ^{25}Mg , ^{19}F

رغد عماد محمود و غيث نعمة فليح

قسم الفيزياء، كلية العلوم، جامعة بغداد، بغداد، العراق

الخلاصة

لقد تم توليد مؤثر الكثافة النووية الفعال ذو صيغة الجسيمين لنظام النوية النقطية الذي يتعامل مع نويات النواة بمثابة جسيمات نقطية ليس لها حجم. اضيفت الى هذا المؤثر دالة ارتباط الجسيمين (التي تأخذ بنظر الاعتبار تأثير كل من القوة التناظرية قصيرة المدى (SRC) و قوة tensor التجاذبية الطويلة المدى (Tc) المتولدتان في التفاعلات النوية - النوية) للنوى ^{27}Al , ^{19}F و ^{25}Mg . اظهرت هذه الدراسة بان ادخال دالة ارتباط الجسيمتين ضمن المدى القصير (SRC) في الحسابات يؤدي الى زيادة احتمالية انتقال البروتونات من مركز النواة باتجاه سطحها يرافقه نقصان واضح في الجزء المركزي من توزيعات كثافة الشحنة النووية 2BCDD's و زيادة طفيفة في الجزء الذيلي منه وبالتالي يؤدي ذلك إلى زيادة القيم المحسوبة لجذر معدل مربع نصف القطر $\langle r^2 \rangle^{1/2}$. وهذا سيجعل النواة اقل صلادة من الحالة التي كانت فيها النواة بدون تأثير الداله (SRC). اظهرت هذه الدراسة ايضا بان تأثير الداله (TC) على كل من 2BCDD's و $\langle r^2 \rangle^{1/2}$ يعاكس تماما تأثير الداله (SRC) عليهما.

Introduction

The charge density distributions (CDD) is the most important quantities in the nuclear structure which was well studied experimentally over a wide range of nuclei. This interest in CDD is related to the basic bulk nuclear

characteristics such as the shape and size of nuclei, their binding energies, and other quantities which are connected with the CDD. Besides, the density distribution is an important object for experimental and theoretical investigations since it plays the role of

a fundamental variable in nuclear theory [1]. The inclusion of short-range and tensor correlation effects is rather a complicated problem especially for the microscopic theory of nuclear structure.

Several methods were proposed to treat complex tensor forces and to describe their effects on the nuclear ground state [2, 3].

A simple phenomenological method for introducing dynamical short range and tensor correlations has been introduced by Dellagiacomma et al. [4]. In that method, two – body correlation operator is introduced to act on the wave function of a pair of particles. A similar correlation operator was proposed earlier by Da Proveidencia and Shakin [5] as well as Malecki and Picchi [6] for describing the short – range correlation effects.

The effect of the short range correlations due to the repulsive part of two-body interaction on the charge form factor of several p-shell nuclei has been analyzed in detail [7] with an independent particle model (IPM) generated in the harmonic oscillator (HO) well [8,9]. In Ref. [7], it was shown that the high-momentum parts ($q > 3 \text{ fm}^{-1}$) of the form factors calculated with and without correlations behave in completely different ways, which indicates that electron scattering at high momentum transfer could give useful information on the short-range correlations. Hamoudi et al. [10] had studied an effective two-body density operator for point nucleon system folded with two-body correlation functions, which take account of the effect of the strong short range repulsion and the strong tensor force in the nucleon-nucleon forces, is produced and used to study the ground state two-body charge density distributions and elastic electron scattering form factors $F(q)$ for $2s - 1d$ shell nuclei with $Z = N$ (such as ^{20}Ne ,

^{24}Mg , ^{28}Si and ^{32}S nuclei). Hamoudi et al. [11] studied the effects of short range correlation and occupation probabilities of single particle orbits for various closed and open shell nuclei with $N=Z$. Hamoudi et al. [12] studied the NMD for the ground state and elastic electron scattering form factors in the framework of the coherent fluctuation model and expressed in terms of the weight function (fluctuation function). The aim of the present work is to study the effects of short range correlations and tensor correlations on the ground state two body charge density distributions, root mean square charge radii and elastic electron scattering form factors for ^{19}F , ^{25}Mg and ^{27}Al nuclei.

Theory

The one body density operator

$$\hat{\rho}^{(1)}(\vec{r}) = \sum_{i=1}^A \delta(\vec{r} - \vec{r}_i) \quad (1)$$

can be transformed into a two-body density form as [1].

$$\hat{\rho}^{(1)}(\vec{r}) \Rightarrow \hat{\rho}^{(2)}(\vec{r})$$

i.e.

$$\sum_{i=1}^A \delta(\vec{r} - \vec{r}_i) \equiv \frac{1}{2(A-1)} \sum_{i \neq j} \left\{ \delta(\vec{r} - \vec{r}_i) + \delta(\vec{r} - \vec{r}_j) \right\} \quad (2)$$

where $\delta(\vec{r} - \vec{r}_i)$: is the Dirac delta function, A is the nucleon number

In fact, a further useful transformation can be made which is that of the

coordinates of two – particles, \vec{r}_i and

\vec{r}_j , to be in terms of that relative

\vec{r}_{ij} and center – of – mass \vec{R}_{ij} coordinates [13].

i.e.

$$\vec{r}_{ij} = \frac{1}{\sqrt{2}} (\vec{r}_i - \vec{r}_j) \quad (3-a)$$

$$\vec{R}_{ij} = \frac{1}{\sqrt{2}}(\vec{r}_i + \vec{r}_j) \quad (3-b)$$

$$\vec{r}_j = \frac{1}{\sqrt{2}}(\vec{R}_{ij} - \vec{r}_{ij}) \quad (3-d)$$

Subtracting and adding (3-a) and (3-b) we obtain

Introducing Eqs.(3-c) and (3-d) into Eq. (2) yields

$$\vec{r}_i = \frac{1}{\sqrt{2}}(\vec{R}_{ij} + \vec{r}_{ij}) \quad (3-c)$$

$$\hat{\rho}^{(2)}(\vec{r}) = \frac{1}{2(A-1)} \sum_{i \neq j} \left\{ \delta \left[\vec{r} - \frac{1}{\sqrt{2}}(\vec{R}_{ij} + \vec{r}_{ij}) \right] + \delta \left[\vec{r} - \frac{1}{\sqrt{2}}(\vec{R}_{ij} - \vec{r}_{ij}) \right] \right\} \quad (4)$$

Eq. (4) may be written as

$$\begin{aligned} \hat{\rho}^{(2)}(\vec{r}) &= \frac{1}{2(A-1)} \sum_{i \neq j} \left\{ \delta \left[\frac{1}{\sqrt{2}}(\sqrt{2}\vec{r} - \vec{R}_{ij} - \vec{r}_{ij}) \right] + \delta \left[\frac{1}{\sqrt{2}}(\sqrt{2}\vec{r} - \vec{R}_{ij} + \vec{r}_{ij}) \right] \right\} \\ &= \frac{2\sqrt{2}}{2(A-1)} \sum_{i \neq j} \left\{ \delta \left[\sqrt{2}\vec{r} - \vec{R}_{ij} - \vec{r}_{ij} \right] + \delta \left[\sqrt{2}\vec{r} - \vec{R}_{ij} + \vec{r}_{ij} \right] \right\} \end{aligned} \quad (5)$$

$$\hat{\rho}^{(2)}(\vec{r}) = \frac{\sqrt{2}}{(A-1)} \sum_{i \neq j} \left\{ \delta \left[\sqrt{2}\vec{r} - \vec{R}_{ij} - \vec{r}_{ij} \right] + \delta \left[\sqrt{2}\vec{r} - \vec{R}_{ij} + \vec{r}_{ij} \right] \right\} \quad (6)$$

where the following identities [14] have been used

or closed shell nuclei with $N=Z$, the two – body charge density operator can be deduced from Eq. (6) as

$$\delta(ax) = \frac{1}{|a|} \delta(x) \text{ (for one – dimension)}$$

$$\hat{\rho}_{ch}^{(2)}(\vec{r}) = \frac{1}{2} \hat{\rho}^{(2)}(\vec{r})$$

$$\delta(a\vec{r}) = \frac{1}{|a^3|} \delta(\vec{r}) \text{ (for three– dimension)}$$

i.e

$$\hat{\rho}_{ch}^{(2)}(\vec{r}) = \frac{\sqrt{2}}{2(A-1)} \sum_{i \neq j} \left\{ \delta \left[\sqrt{2}\vec{r} - \vec{R}_{ij} - \vec{r}_{ij} \right] + \delta \left[\sqrt{2}\vec{r} - \vec{R}_{ij} + \vec{r}_{ij} \right] \right\} \quad (7)$$

Finally, an effective two-body charge density operator (to be used with uncorrelated wave functions) can be produced by folding the operator of

Eq.(7) with the two-body correlation functions \tilde{f}_{ij} as

$$\hat{\rho}_{eff}^{(2)}(\vec{r}) = \frac{\sqrt{2}}{2(A-1)} \sum_{i \neq j} \tilde{f}_{ij} \left\{ \delta \left[\sqrt{2}\vec{r} - \vec{R}_{ij} - \vec{r}_{ij} \right] + \delta \left[\sqrt{2}\vec{r} - \vec{R}_{ij} + \vec{r}_{ij} \right] \right\} \tilde{f}_{ij} \quad (8)$$

In the present work, a simple model form of the two-body full correlation

operators of ref. [15] will be adopted, i.e

$$\tilde{f}_{ij} = f(r_{ij})\Delta_1 + f(r_{ij})\{1 + \alpha(A)S_{ij}\}\Delta_2 \tag{9}$$

It is obvious that this equation includes two types of correlations:

(a) The two-body short rang correlations (SRC's) presented in the first term of Eq. (9) and denoted by $f(r_{ij})$. Here Δ_1 is a projection operator onto the space of all two-body

$$f(r_{ij}) = \begin{cases} 0 & \text{for } r_{ij} \leq r_c \\ 1 - \exp\{-\mu(r_{ij} - r_c)^2\} & \text{for } r_{ij} > r_c \end{cases} \tag{10}$$

where r_c (in fm) is the radius of a suitable hard core and $\mu = 25 \text{ fm}^{-2}$ [15] is a correlation parameter.

(b) The two-body tensor correlations (TC's) presented in the second term of Eq.(9) are induced by the strong tensor component in the nucleon-nucleon force and they are of longer range. Here Δ_2 is a projection operator onto the 3S_1 and 3D_1 states only. However, Eq.(9) can be rewritten as

$$\tilde{f}_{ij} = f(r_{ij}) \sum_{\gamma} \{1 + \alpha_{\gamma}(A)S_{ij}\}\Delta_{\gamma} \tag{11}$$

where the sum γ , in Eq.(11), is over all reaction channels, S_{ij} is the usual tensor operator, formed by the scalar product of a second-rank operator in

functions with the exception of 3S_1 and 3D_1 states.

In fact, the SRC's are central functions of the separation between the pair of particles which reduce the two-body wave function at short distances, where the repulsive core forces the particles apart, and heal to unity at large distance where the interactions are extremely weak. A simple model form of two-body SRC's is given by [15]

intrinsic spin space and coordinate space and is defined by

$$S_{ij} = \frac{3}{r_{ij}^2} (\vec{\sigma}_i \cdot \vec{r}_{ij})(\vec{\sigma}_j \cdot \vec{r}_{ij}) - \vec{\sigma}_i \cdot \vec{\sigma}_j \tag{12}$$

while $\alpha_{\gamma}(A)$ is the strength of tensor correlations and it is non zero only in the $^3S_1 - ^3D_1$ channels.

The ground state two body charge density distribution $\rho_{ch}(r)$ is given by the expectation.

Value of the effective two-body charge density operator of Eq. (8) and expressed as

$$\begin{aligned} \rho_{ch}(r) &\equiv \langle \Psi | \hat{\rho}_{eff}^{(2)}(\vec{r}) | \Psi \rangle \\ &= \sum_{i(j)} \langle i j | \hat{\rho}_{eff}^{(2)}(\vec{r}) [|ij\rangle - |ji\rangle] \end{aligned} \tag{13}$$

where the two particle wave function is given by [16]

$$|ij\rangle = \sum_{J_M} \sum_{T_M} \langle j_i m_i j_j m_j | JM_J \rangle \langle t_i m_{t_i} t_j m_{t_j} | TM_T \rangle | (j_i j_j) JM_J \rangle | (t_i t_j) TM_T \rangle \tag{14}$$

It is important to indicate that our effective two body charge density

operator of Eq. (8) is constructed in terms of relative and center of mass

coordinates, thus the space-spin part $|(j_i j_j) JM_J\rangle$ of two-particle wave function constructed in *jj*-coupling scheme must be transformed in terms of relative and center of mass coordinates.

The nuclear mean square charge radius $\langle r^2 \rangle^{1/2}$ is defined by [16]

$$\langle r^2 \rangle = \frac{4\pi}{Z} \int_0^\infty \rho_{ch}(r) r^4 dr \quad (15)$$

Elastic electron scattering form factor from spin zero nuclei ($J = 0$), can be determined by the ground – state charge density distributions (CDD). In the Plane Wave Born Approximation (PWBA), the incident and scattered electron waves are considered as plane waves and the CDD is real and spherical symmetric, therefore the form factor is simply the Fourier transform of the CDD. Thus [17, 18]

$$F(q) = \frac{4\pi}{Z} \int_0^\infty \rho_o(r) j_0(qr) r^2 dr \quad (16)$$

where $\rho_o(r)$ is the ground state 2BCDD of Eq (16).

$j_0(qr) = \sin(qr)/(qr)$ is the zeroth order of the spherical Bessel function

$$F(q) = \frac{4\pi}{qZ} \int_0^\infty \rho_o(r) \sin(qr) r dr F_{fs}(q) F_{cm}(q) \quad (20)$$

In the limit of $q \rightarrow 0$, the target will be considered as a point particle, and from Eq.(20), the form factor of this target nucleus is equal to unity, i.e. $F(q \rightarrow 0)=1$. The elastic longitudinal electron scattering form factor with the inclusion of the effect of the two-body TC's in light nuclei can now be obtained by introducing the ground state 2BCDD of Eq.(8) into Eq.(20). We also wish to mention that we have written all computer programs needed

and q is the momentum transfer from the incident electron to the target nucleus. Eq. (16) may be expressed as

$$F(q) = \frac{4\pi}{qZ} \int_0^\infty \rho_o(r) \sin(qr) r dr \quad (17)$$

Inclusion of the finite nucleon size correction $F_{fs}(q)$ and the center of mass correction $F_{cm}(q)$ in our calculations requires multiplying the form factor of Eq. (17) by these corrections. $F_{fs}(q)$ is considered as free nucleon form factor and assumed to be the same for protons and neutrons. This correction takes the form [19].

$$F_{fs}(q) = e^{-0.43q^2/4} \quad (18)$$

The correction $F_{cm}(q)$ removes the spurious state arising from the motion of the center of mass when shell model wave function is used and given by [19].

$$F_{cm}(q) = e^{q^2 b^2 / 4A} \quad (19)$$

where A is the nuclear mass number. Introducing these corrections into Eq. (17), we obtain

in this study using Fortran-90 languages.

Results and discussion

The calculations for the ground state two body charge density distributions (2BCDD's) $\rho_{ch}(r)$, the root mean square charge radii $\langle r^2 \rangle^{1/2}$ and elastic electron scattering form factors $F(q)$'s are carried out for ^{19}F , ^{25}Mg and ^{27}Al nuclei. A choice for the single value of the hard core radius $r_c = 0.4 \text{ fm}$ is adopted for all considered

nuclei. The strengths of the tensor correlations $\alpha(A)$ are determined by fitting the calculated $\langle r^2 \rangle^{1/2}$ with those of experimental data. All parameters required in the calculations of $\rho_{ch}(r)$, $\langle r^2 \rangle^{1/2}$ and $F(q)$'s, such as the harmonic oscillator spacing parameter, $\hbar\omega$, the occupation probabilities, η 's, of the states and $\alpha(A)$, are presented in Table 1. It is clear from this table that the result for $\alpha(A)$ is a decreasing function with

increasing the mass number A .

Besides, the calculated $\langle r^2 \rangle_{r_c=0.4, \alpha \neq 0}^{1/2}$ including both effects of SRC (with $r_c = 0.4 \text{ fm}$) and TC (with $\alpha(A) \neq 0$), are in very good agreement with those of experimental data [20]. The results for the dependence of $\rho_{ch}(r)$ (in fm^{-3}) on r (in fm) for ^{19}F , ^{25}Mg and ^{27}Al nuclei are displayed in Figs.1.

Table 1: Parameters which have been used in the calculations of the present work for the 2BCDD's, and elastic longitudinal $F(q)$'s of all nuclei under study.

Nucleus	^{19}F	^{25}Mg	^{27}Al
$\hbar\omega$	11	11.75	12.75
$\eta_{1S_{1/2}}$	1	1	1
$\eta_{1P_{3/2}}$	0.5	0.25	0.25
$\eta_{1P_{1/2}}$	1	1	1
$\eta_{1d_{5/2}}$	0.1666	0.8333	1
$\eta_{2S_{1/2}}$	0.5	1	1
$\alpha(A)$	0.2	0.198	0.197
$\langle r^2 \rangle_{r_c=0, \alpha=0}^{1/2}$	2.891439	3.068455	3.015337
$\langle r^2 \rangle_{r_c=0.4, \alpha \neq 0}^{1/2}$	2.862643	3.027741	2.977019
$\langle r^2 \rangle_{\text{ex.}}^{1/2}$ [20]	2.901144	3.109868	3.061974

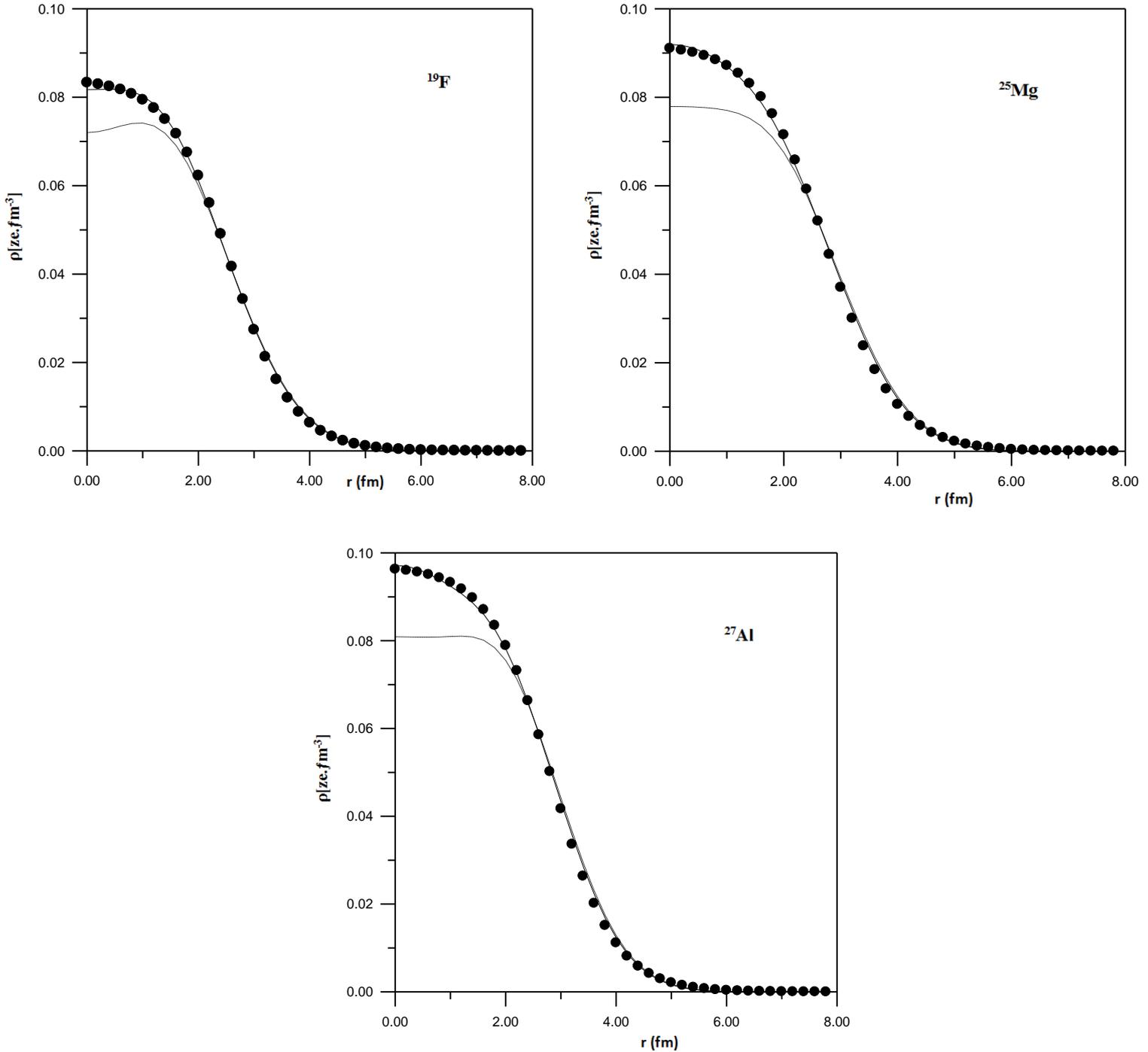


Fig.1: Dependence of the 2BCDD on r (fm) for ^{19}F , ^{25}Mg and ^{27}Al nuclei. The dotted symbols are the experimental data of Ref [20].

In Fig.1, the dashed and solid distributions are the calculated $\rho_{ch}(r)$ of ^{19}F , ^{25}Mg and ^{27}Al nuclei without effects ($r_c=0$ and $\alpha(A)=0$) and with the effects of SRC and TC ($r_c = 0.4$ fm and $\alpha(A) \neq 0$) included, respectively. These distributions are compared with those of experimental data [20], denoted by dotted symbols. In ^{19}F ,

^{25}Mg and ^{27}Al nuclei, the dashed distributions deviate from the experimental data especially at small r . Introducing the effects of SRC and TC tends to remove these deviations from the region of small r as seen in the solid distributions. However, both the dashed and solid distributions overestimate the data at the central

region of $\rho_{ch}(r)$ whereas beyond this region they agree very well with the data.

Elastic electron scattering charge form factors $F(q)$ for the ^{19}F , ^{25}Mg and ^{27}Al nuclei are calculated using the ground state two body charge density distributions of Eq. (13) in Eq. (20). In

Fig.2, the calculated $F(q)$ s are compared with those of experimental data for ^{19}F , ^{25}Mg and ^{27}Al nuclei. The dashed and solid distributions are the calculated $F(q)$'s without and with the effects of SRC and TC, respectively, while the dotted symbols are those of experimental data.

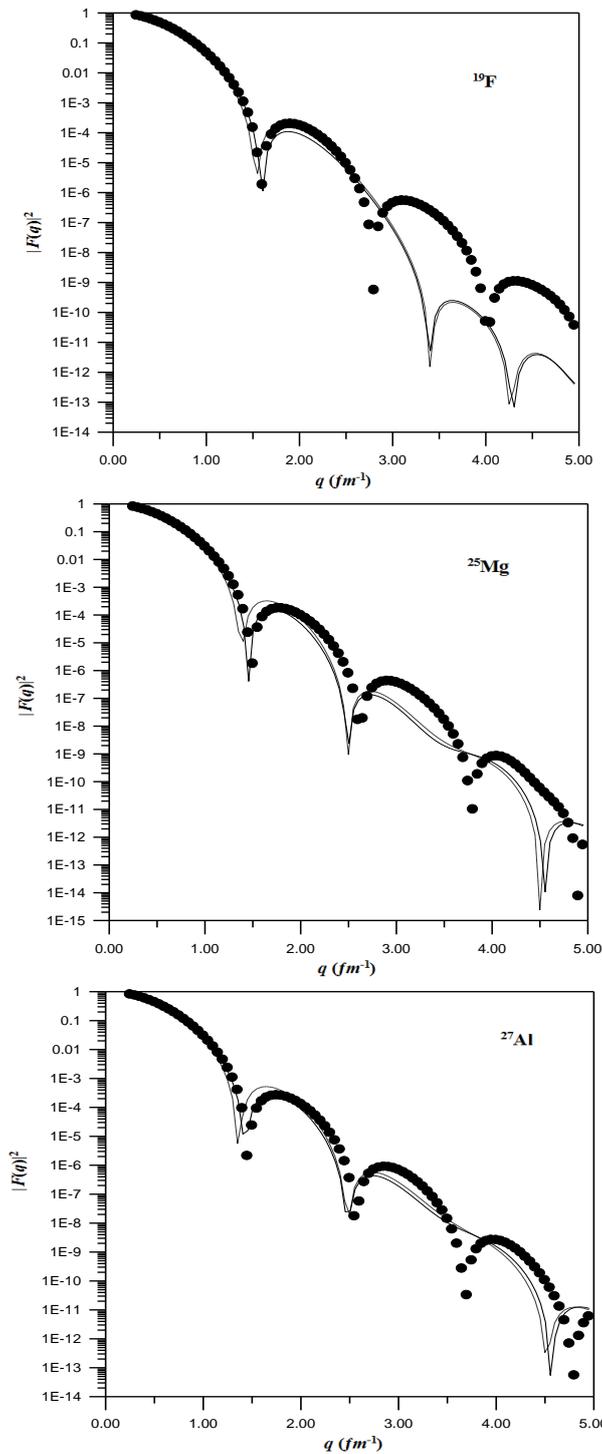


Fig.2: Dependence of the $F(q)$ s on q (fm^{-1}) for ^{19}F , ^{25}Mg and ^{27}Al nuclei. The dotted symbols are the experimental data of Ref. [20] for ^{19}F , ^{25}Mg and ^{27}Al .

In ^{19}F nucleus, the first diffraction minimum and first maximum which are known from the experimental data [20] are very well reproduced by the dashed and solid curves. In general, the calculated $F(q)$'s are in very good accordance with the data up to momentum transfer $q \approx 2.4 \text{ fm}^{-1}$. For higher q , the calculated form factors are in disagreement with the data, where the second diffraction minimum observed in $q=2.5 \text{ fm}^{-1}$.

In ^{25}Mg nucleus, It is noticed from the figure the dashed and solid curves are in reasonable agreement with those of experimental data [20] throughout the range of momentum transfer $q \leq 1.5 \text{ fm}^{-1}$. It is noted that the effect of correlations begin at the region of $q > 2.3 \text{ fm}^{-1}$, where the solid curve deviates from the dashed curve at this region of q .

In ^{27}Al nucleus, the dashed and solid curves are in good agreement with the experimental data up to momentum transfer of $q = 2.3 \text{ fm}^{-1}$, and it underestimates clearly these data at the region of $q > 2.3 \text{ fm}^{-1}$. It is so clear that the location of the third observed diffraction minimum is not reproduced in the correct place by the dashed and solid curves. It is noted that the effect of correlations begin at the region of $q > 2.5 \text{ fm}^{-1}$.

Conclusions

This study shows that the two-body tensor correlations exhibit a mass dependence due to the Strength parameter $\alpha(A)$ while the two-body short range correlations do not exhibit this dependency. Including the effect of *SRC* alone increases the probability of transferring the protons from the central region of $\rho_{ch}(r)$ towards its tail (i.e., the nucleus becomes less rigid than the case when there is no *SRC*) and then increases the calculated $\langle r^2 \rangle^{1/2}$. It is found that the effect of

TC on $\rho_{ch}(r)$ and $\langle r^2 \rangle^{1/2}$ is in opposite direction to that of *SRC*.

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